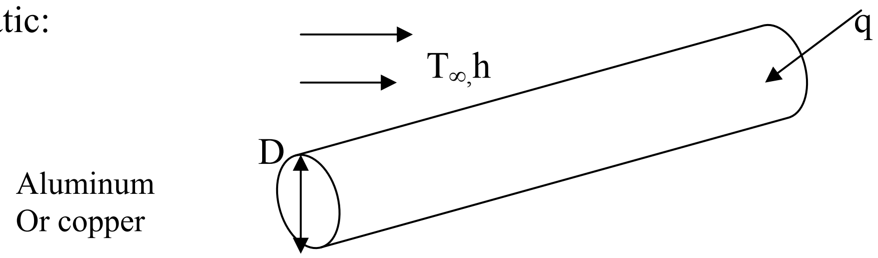
Problem 1:

A long, circular aluminium rod attached at one end to the heated wall and transfers heat through convection to a cold fluid.
(a) If the diameter of the rod is triples, by how much would the rate of heat removal change?
(b) If a copper rod of the same diameter is used in place of aluminium, by how much would the rate of heat removal change?

Known: long, aluminum cylinder acts as an extended surface.

Find: (a) increase in heat transfer if diameter is tripled and (b) increase in heat transfer if copper is used in place of aluminum.

Schematic:



Assumptions: (1) steady-state conditions, (2) one-dimensional conduction, (3) constant properties, (4) uniform convection coefficient, (5)rod is infinitely long.

Properties: aluminum (pure): k=240W/m. K;
copper (pure): k=400W/m. K

Analysis: (a) for an infinitely long fin, the fin rate is

$$qr = M = (hpkA_c)^{\frac{1}{2}}\theta_b$$

$$q_f = (h\pi Dk\pi D^2/4)^{\frac{1}{2}}\theta_b = \frac{\pi}{2}(hk)^{\frac{1}{2}}D^{\frac{3}{2}}\theta_b$$

Where P=πD and $A_c=\pi D^2/4$ for the circular cross-section. Note that $q_f \approx D^{3/2}$. Hence, if the diameter is tripled,

$$\frac{q_{f(3D)}}{q_{f(D)}} = 3^{\frac{3}{2}} = 5.2$$

And there is a 520 % increase in heat transfer.

(b) in changing from aluminum to copper, since $q_f \approx k^{1/2}$, it follows that

$$\frac{q_f(Cu)}{q_f(Al)} = \left(\frac{k_{Cu}}{k_{Al}}\right)^{\frac{1}{2}} = \left(\frac{400}{240}\right)^{\frac{1}{2}} = 1.29$$

And there is a 29 % increase in the heat transfer rate.

Comments: (1) because fin effectiveness is enhanced by maximum $P/A_c = 4/D$. the use of a larger number of small diameter fins is preferred to a single large diameter fin.

(2) From the standpoint of cost and weight, aluminum is preferred over copper.

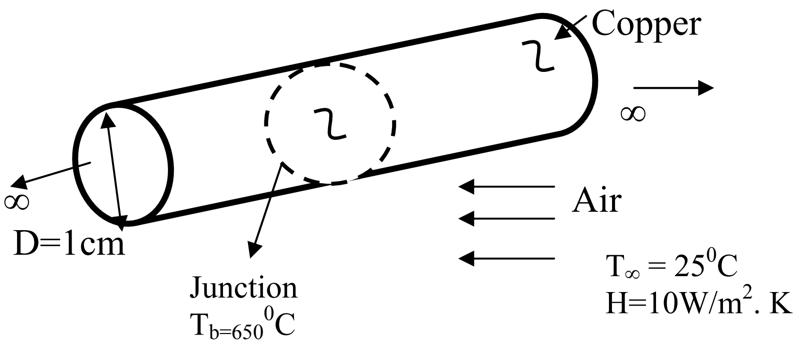
Problem 2:

Two long copper rods of diameter D=1 cm are soldered together end to end, with solder having melting point of 650°C. The rods are exposed to air at 250°C with a convection heat transfer coefficient of 10W/m^2. K. Estimate the minimum electrical power input needed for the soldering to occur?

Known: Melting point of solder used to join two long copper rods.

Find: Minimum power needed to solder the rods.

Schematic:



Assumptions: (1) steady-state conditions, (2) one-dimensional conduction along the rods, (3) constant properties, (4) no internal heat generation, (5) negligible radiation exchange with surroundings, (6) uniform, h and (7) infinitely long rods.

Properties: copper $\bar{T}$ = (650+25)^{0}C ≈ 600K: k=379 W/m.K

Analysis: the junction must be maintained at 650^{0}C while energy is transferred by conduction from the junction (along both rods). The minimum power is twice the fin heat rate for an infinitely long fin,

$$q_{min} = 2q_f = 2(hpkA_c)^{\frac{1}{2}}(T_b - T_\infty)$$

substituting numerical values,

$$q_{min} = 2\left(10\frac{W}{m^2.K}(\pi \times 0.01m)\left(379\frac{W}{m.K}\right)\frac{\pi}{4}(0.01m)^2\right)^{\frac{1}{2}}(650 - 25)^0C.$$

therefore,

$$q_{min} = 120.9W$$

Comments: radiation losses from the rod are significant, particularly near the junction, thereby requiring a larger power input to maintain the junction at 650^0C.

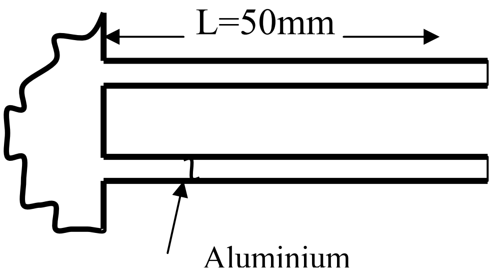
Problem 3:

Heat transfer enhancement is required from a heated plane wall. It is proposed that aluminium fins of rectangular profile are attached to the wall. The fins are 50mm long, 0.5mm thick, and are equally spaced at a distance of 4mm. The fin spacing is 250fins per metre. The heat transfer coefficient associated with the unfinned plane wall is $40 W/m^2.K$, while that associated with the fin surface is $30 W/m^2.K$. Find the percentage increase in heat transfer resulting from use of fins.

Known: Dimensions and number of rectangular aluminum fins. Convection coefficient with and without fins.

Find: percentage increase in heat transfer resulting from use of fins.

Schematic:



$N=250 m^{-1}$
W=width
$h_w = 30 W/m^2.K$(with fins)
$h_{wo}=40 W/m^2.K$(without fins)

Assumptions: (1) steady-state conditions, (2) one-dimensional conduction, (3) constant properties, (4) negligible fin contact resistance, (6) uniform convection coefficient.

Properties: Aluminum, pure: $k \approx 240 W/m.K$

Analysis: evaluate the fin parameters

$$L_c = L + \frac{t}{2} = 0.0505\text{m}$$

$$A_p = L_c t = 0.0505\text{m} \times 0.5 \times 10^{-3}\text{m} = 25.25 \times 10^{-6}\text{m}^2$$

$$L_c^{3/2}(h_w / KA_P)^{1/2} (0.0505\text{m})^{3/2} \left(\frac{30\text{W}/\text{m}^2.\text{K}}{240\text{W}/\text{m.K} \times 25.25 \times 10^{-6}\text{m}^2} \right)^{1/2}$$

$$L_c^{3/2}(h_w / KA_P)^{1/2} = 0.80$$

it follows that, $\eta_f = 0.72$. hence

$$q_f = \eta_f q_{max} = 0.72 h_w 2wL\theta_b$$

$$q_f = 0.72 \times 30\text{W}/\text{m}^2.\text{K} \times 2 \times 0.05\text{m} \times (w\theta_b) = 2.16\text{W}/\text{m}/\text{K}(w\theta_b)$$

With the fins, the heat transfer from the walls is

$$q_w = Nq_f + (1 - Nt)wh_w\theta_b$$

$$q_w = 250 \times 2.16 \frac{\text{W}}{\text{m.K}}(w\theta_b) + (1\text{m} - 250 \times 5 \times 10^{-4}\text{m}) \times 30\text{W}/\text{m}^2.\text{K}(w\theta_b)$$

$$q_w = (540 + 26.63)\frac{\text{W}}{\text{m.K}}(w\theta_b) = 566w\theta_b$$

Without the fins, $q_{wo} = h_{wo} 1\text{m} \times w\theta_b = 40w\theta_b$.

Hence the percentage increases in heat transfer is

$$\frac{q_w}{q_{wo}} = \frac{566w\theta_b}{40w\theta_b} = 14.16 = 1416\%$$

Comments: If the finite fin approximation is made, it follows that $q_f = (hPkA_{c)})^{1/2}\theta_b = [h_w 2wkwt]^{1/2}\theta_b = (30*2*240*5*10^{-4})^{1/2}\, w\theta_b = 2.68w\theta_b$. Hence q_f is overestimated.