Exercises for Module 3

of

Micro and Smart Systems NPTEL Course

(The following problems are taken from the instructors' book entitled "Micro and Smart Systems", John Wiley, 2012.)

3.1 Figure 3.1 shows three variants of a bar of square cross section in a resonant-mode force sensor. An axial force of 100 μN acts on it in all the three cases. Find the elongation of the bar and the maximum stress and strain in all three cases if the length of the bar is 200 μm. Take Young's modulus of silicon to be 150 GPa.

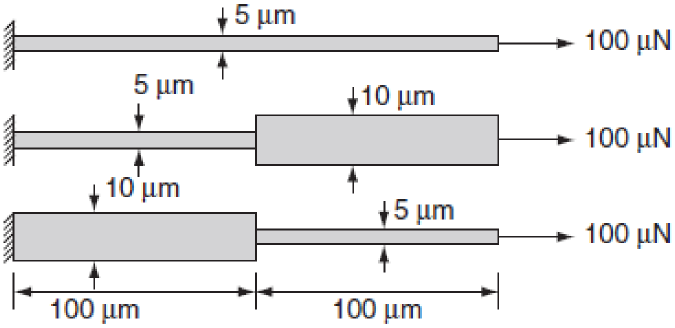

Fig. 3.1 A bar under axial load with three variant shapes

3.2 Suspension of an accelerometer is schematically shown in Figure 3.2. There are three beams on either side of the square proof mass of 50 μg mass. The beams are identical. They are 150 μm long and have an in-plane width of 8 μm and an out-of-plane thickness of 2 mm. They are made of polysilicon whose Young's modulus is 169 GPa.

(a) How much does the proof mass move in the x-direction for 1 g (9.8 m/s^2) acceleration in that direction?

(b) How much does the proof mass move in the y-direction for 1 g (9.8 m/s^2) acceleration in that direction?

(c) How much does the proof mass move in the x-direction and y-direction for 1 g (9.8 m/s^2) acceleration at an angle of 45° to the x-direction?

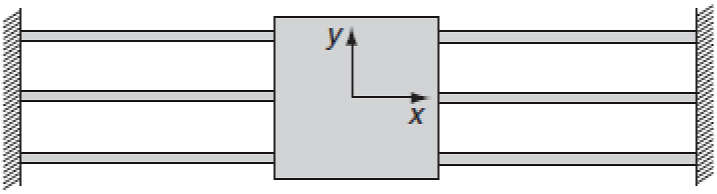

Fig. 3.2 A six-beam suspension of a proof-mass in an accelerometer

3.3 Figure 3.3 shows a spring steel cantilever beam (Young's modulus = 210 GPa) over which there is a piezopatch (Young's modulus = 300 GPa). The lengths of the beam and the piezopatch are shown in the figure. The in-plane width (25 μm) and out-of-plane thickness (5 μm) of the beam and piezopatch are the same. If the piezopatch contracts axially by 1%, compute the deflection of the free tip of the spring steel beam in the axial and transverse directions.

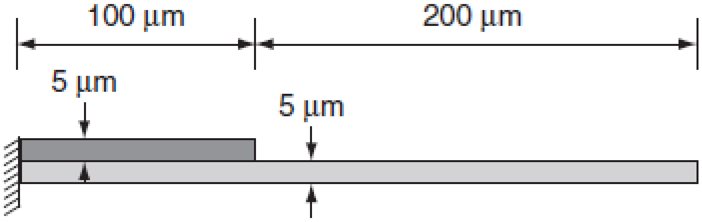

Fig. 3.3 A spring steel beam with a piezopatch over one-third of its length

3.4 Figure 3.4(a) shows a rigid mass suspended from one cantilever and Figure 3.4(b) shows the same with two cantilevers. If the same force of 1 mN acts in both cases, compute the deflection of the mass in the direction of the force. Sketch the deformed profiles schematically in both the cases. The beams are 250 μm long and have out-of-plane thickness of 3 μm and in-plane width of 4 μm. The cross-sections of the beams are rectangular. Take Young's modulus = Y = 169 GPa.

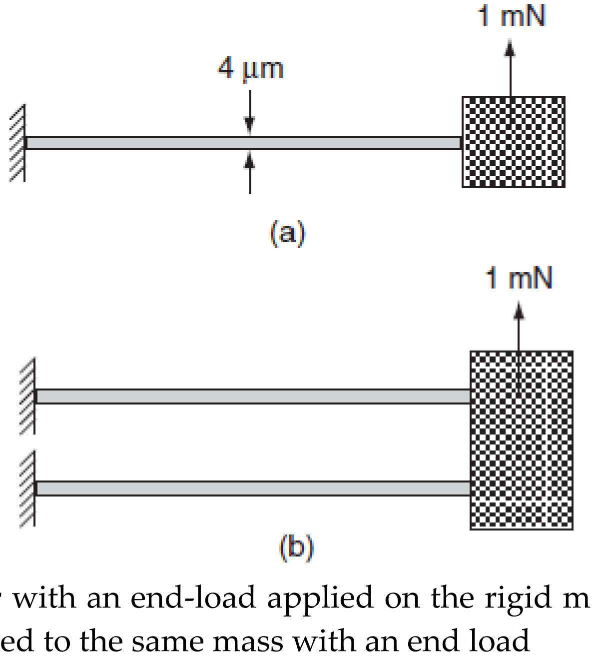

Fig. 3.4 (a) A cantilever with an end-load applied on the rigid mass, (b) the case of two cantilever beams attached to the same mass with an end load

3.5 A research lab tried to make a 314 μm long cantilever beam with diamond-like-carbon (DLC) thin-film technology. The beam had rectangular cross section with 15 μm in-plane width and 5 μm out-of-plane thickness. In its released condition, it was noticed that it had curled into a semicircular form. The Young's modulus and Poisson's ratio of DLC were measured to be 100 GPa and 0.3, respectively.

(a) What is the residual stress gradient that caused this curling?

(b) What is the anticlastic curvature along the in-plane width direction? Draw that shape.

3.6 A silicon nitride beam of rectangular cross-section in a microscale flow-meter is fixed at both the ends. It has length 800 μm, in-plane width 10 μm, and out-of-plane thickness 2 μm. There is an aluminum resistor over this beam all along its length, but the thickness of the aluminum beam is 1 μm and its width is only 5 μm. It is placed symmetrically along the in-plane width direction as shown in Figure 3.5. If there is a distributed transverse load of 20 N/m along the length of this composite beam, what is the transverse deflection of its midpoint? (Take the Young's modulus of silicon nitride to be 200 GPa and that of aluminum to be 70 GPa.)

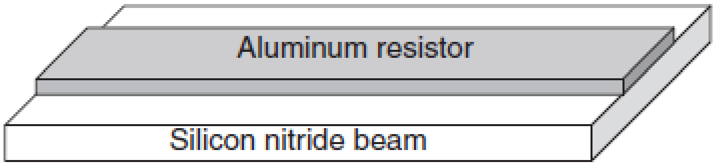

Fig. 3.5 A sandwich beam of aluminum and silicon nitride

3.7 A diamond-shaped micromirror is suspended between two torsional aluminum beams of square cross-section as shown in Figure 3.6. The length of each torsional beam is 100 μm and the side of the cross-section is 2 μm. When there is force in the direction perpendicular to the plane of the diamond-shaped beam at one of its free vertices, it tilts by 5° about the axis of the torsional beams. What is the magnitude of the force? The Young's modulus of aluminum is 70 GPa and Poisson's ratio is 0.25.

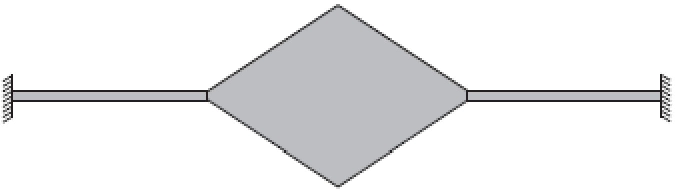

Fig. 3.6 A diamond-shaped micromiror suspended by torsional beams

3.8 In a plane-strain condition, all the strains involving the z-direction are zero. Derive the stress-strain relationship between stresses and strains in the x- and y-directions.

3.9 A permanent magnet is attached at the midpoint of a fixed-fixed beam (see Figure 3.7). Due to an external magnetic field, there is a couple of 100 μN.μm on the magnet about the direction perpendicular to the plane of the magnet. If the beam is 500 μm long and has circular cross section of diameter 10 μm, what and where is the maximum stress induced in the beam?

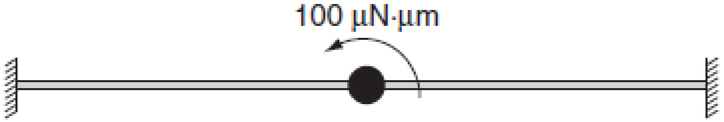

Fig. 3.7 A couple acting at the midpoint of a fixed-fixed beam

4.10 A microvalve has a circular poppet (shown filled in Figure 3.8) suspended by four semicircular beams of radius 50 μm and square cross-section 2 μm/side. Compute the stiffness of the poppet in the direction perpendicular to the plane of the disk. It is made of steel with a Young's modulus of 210 GPa.

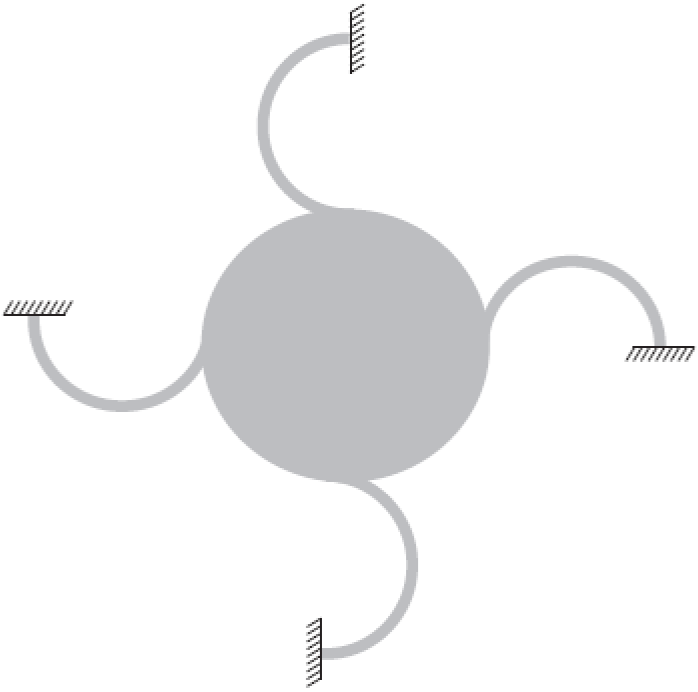

Fig. 3.8 A circular poppet suspended by four semicircular beams