Deflection of beams

Introduction

Deflection of Beams (Solution Method by Direct Integration)

Moment - Area Method for finding Beam Deflections

Introduction

The axis of a beam deflects from its initial position under action of applied forces. Accurate values for these beam deflections are sought in many practical cases: elements of machines must be sufficiently rigid to prevent misalignment and to maintain dimensional accuracy under load; in buildings, floor beams cannot deflect excessively to avoid the undesirable psychological effect of flexible floors on occupants and to minimize or prevent distress in brittle-finish materials; likewise, information on deformation characteristics of members is essential in the study of vibrations of machines as well as of stationary and flight structures.

Top

Deflection of Beams

(Solution Method by Direct Integration)

From Analytic geometry, Curvature of a line,

$$k = \frac{1}{R} = \frac{\dfrac{d^2y}{dx^2}}{\left\{1+\left(\dfrac{d^2y}{dx^2}\right)\right\}^{3/2}}$$

where x and y are co-ordinates of a point on the curve.

For small deflections,

$$k = \frac{d^2y}{dx^2}$$

Since, $\sigma_x = -My/EI$

and $k = -\varepsilon/y,$ and $\varepsilon = \sigma_x/E$

$$k = M/EI$$

$$\therefore M = EI\frac{d^2y}{dx^2}$$

where $M = M_y$

Hence,

$$EI\frac{d^2y}{dx^2} = M(x)$$

Since, $V = \dfrac{dM}{dx}$

$$EI\frac{d^3y}{dx^3} = V(x)$$

Since, $w = \dfrac{dv}{dx}$

$$EI\frac{d^4y}{dx^4} = w(x)$$

Boundary Conditions

Refer figure 5.2.7(a) – (d)

(a) Clamped Support:

$y(x_1) = 0;\ y'(x_1)=0;$

(b) Roller or Pinned Support:

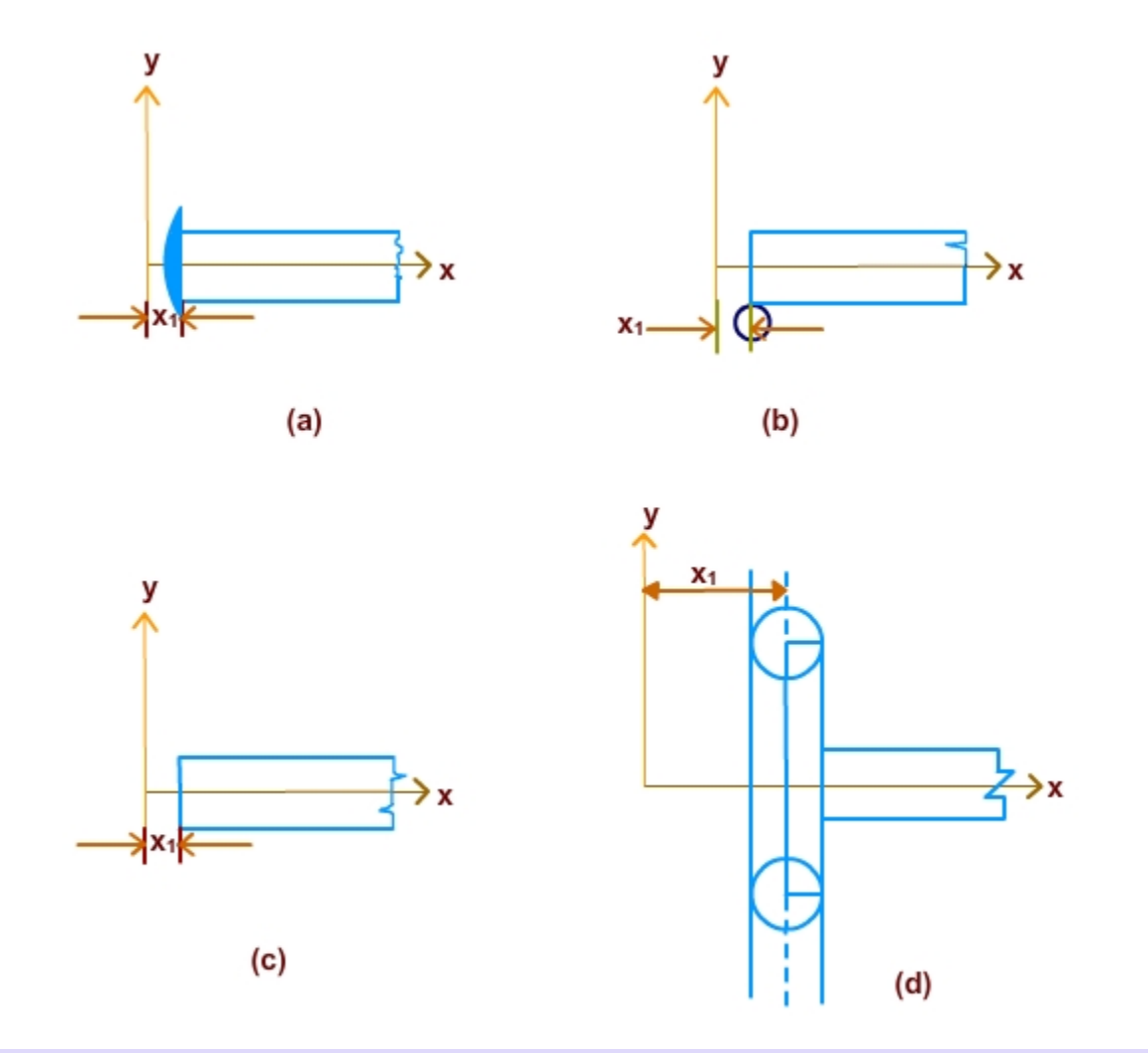
$y(x_1) = 0;\ M(x_1)=0;$

(c) Free end:

$M(x_1) = 0;\ V(x_1) = 0;$

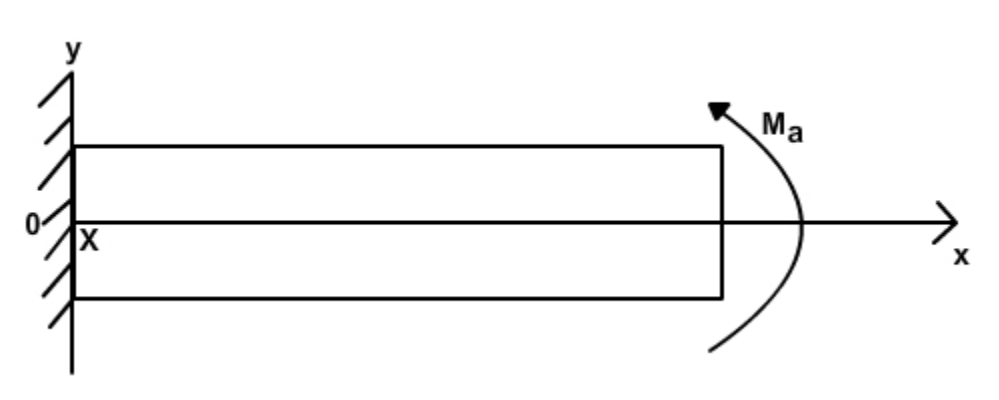
(d) Guided Support:

$y'(x_1) = 0;\ V(x_1)=0;$



Example:

Question: A Cantilever beam is subjected to a bending moment M at the force end. Take flexural rigidity to be constant and equal to EI. Find the equation of the elastic curve.



$$EI\frac{d^2y}{dx^2} = M_a$$

Integrating

$EI\ dy/dx = M_a x + C_1$

at $x = 0$; $dy/dx = 0$

which gives $C_1 = 0$

Integrating again,

$$EIy = \frac{M_a x^2}{2} + C_2$$

$y = 0$; at $x = 0$ gives

$C_2 = 0$

$$\therefore\ y = \frac{M_a x^2}{2EI}$$

which is the equation to the elastic curve.

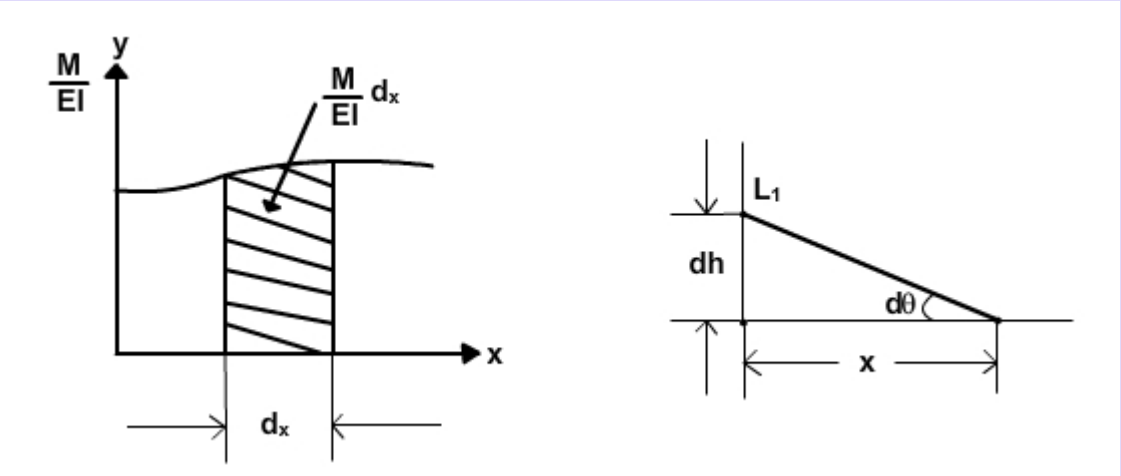
Top

Moment - Area Method for finding Beam Deflections

This method is used generally to obtain displacement and rotation at a single point on a beam.

This method makes use of the Moment - Area theorems given below.

Moment - Area Theorems



Refer above Figure

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$\frac{M}{EI} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

Let $dy/dx = \theta$

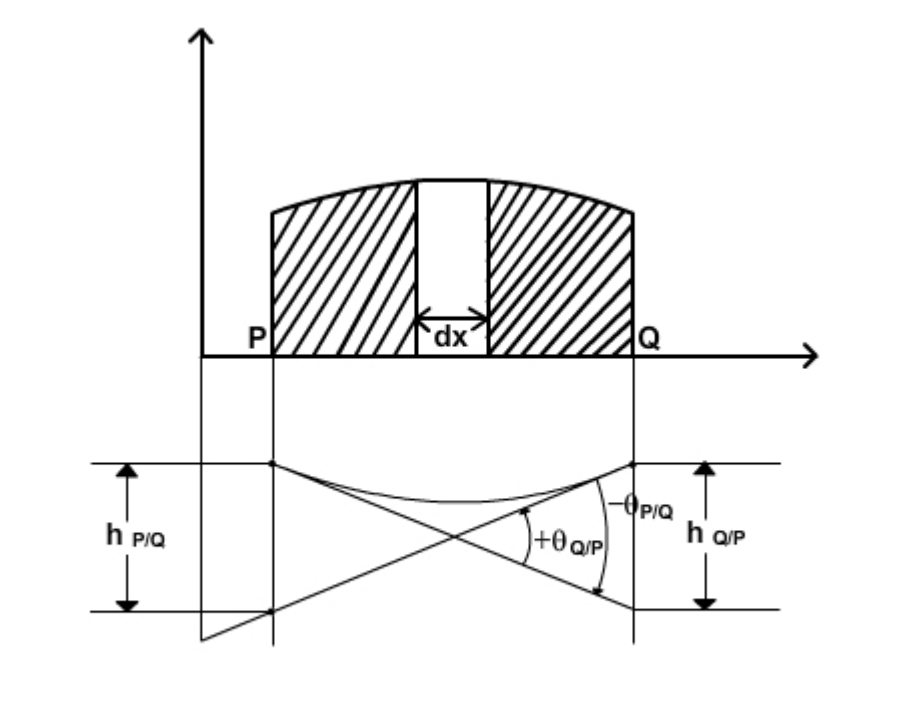
$$\frac{M}{EI} = \frac{d\theta}{dx}$$

$$\int d\theta = \int \frac{M}{EI}\, dx$$

Referring to figure down

$$\therefore\ \theta_{Q/P} = \theta_Q - \theta_P = \int_P^Q \frac{M}{EI}\, dx$$

This is the first moment area theorem, Where P and Q are any two sections on the beam. ie change in angle measured in radians between any two point P and Q on the elastic curve is equal to the M/EI area bounded by the ordinates through P and Q.



Referring to Figure, considering an element of the Elastic Curve,

$dh = x \cdot d\theta$

$$\therefore\ h_{Q/P} = \int_P^Q dh$$

$$= \int_P^Q x d\theta$$

$$= \int_P^Q \frac{M}{EI}\, x\, dx$$

This is the second moment Area theorem.

If $\overline{x_P}$ is the distance of centroid of the bending moment diagram between P and Q from P,

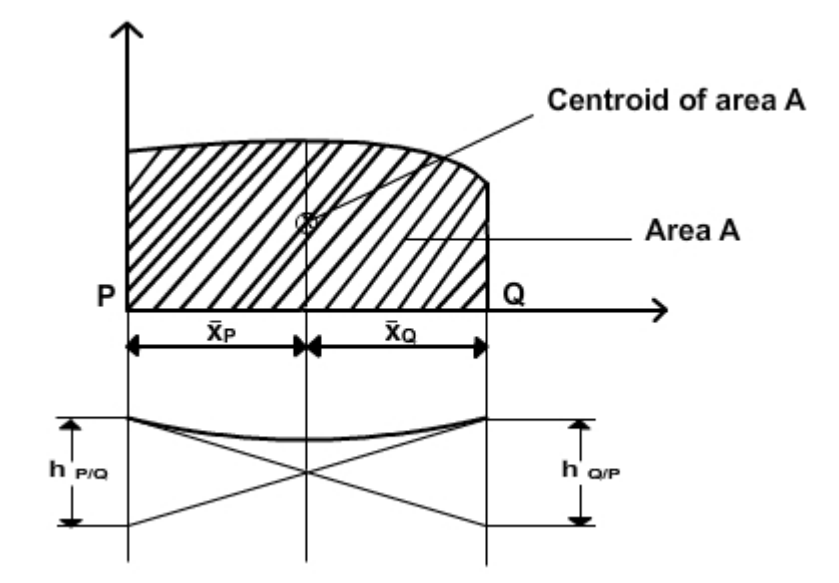
(Refer Figure)

then

$$h_{P/Q} = A\overline{x_P}$$

$$h_{Q/P} = A\overline{x_Q}$$

Here $h_{P/Q}$ is called the tangent deviation of the point P from a tangent at Q.



where A is the area of the BM diagram between P and Q. $\overline{x_p}$ and $\overline{x_Q}$ are as shown in figure.

Top