

Module 2 - GEARS

Lecture 7 - SPUR GEAR DESIGN

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7.1 SPUR GEAR – TOOTH FORCE ANALYSIS

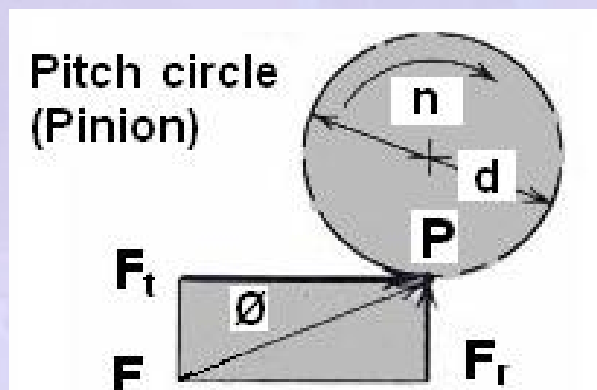


Fig. 7.1 Spur gear – tooth force analysis

As shown in Fig.. 7.1, the normal force F can be resolved into two components; a tangential force F_t which does transmit the power and radial component F_r which does no work but tends to push the gears apart. They can hence be written as,

$$F_t = F \cos \emptyset \quad (7.1)$$

$$F_r = F \sin \emptyset \quad (7.2)$$

From eqn. (7.2),

$$F_r = F_t \tan \emptyset \quad (7.3)$$

The pitch line velocity V , in meters per second, is given as

$$V = \frac{\pi d n}{6000} \quad (7.4)$$

$$W = \frac{F_t V}{1000} \quad (7.5)$$

where d is the pitch diameter of the gear in millimeters and n is the rotating speed in rpm and W power in kW.

7.2 SPUR GEAR - TOOTH STRESSES

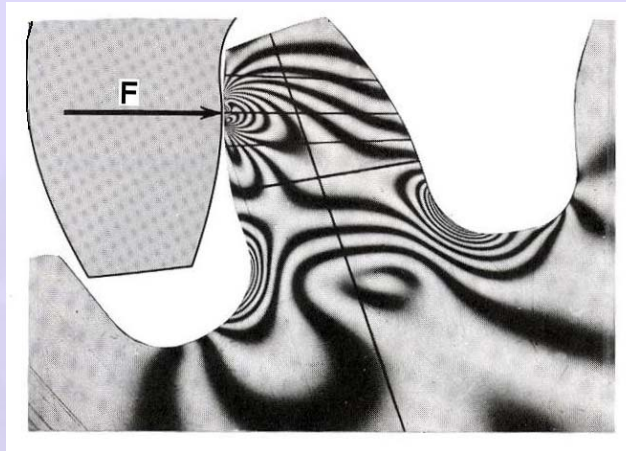


Fig. 7.2 Photo-elastic Model of gear tooth

Stresses developed by Normal force in a photo-elastic model of gear tooth as per Dolan and Broghammer are shown in the Fig.. 7.2. The highest stresses exist at regions where the lines are bunched closest together. The highest stress occurs at two locations:

- A. At contact point where the force F acts
- B. At the fillet region near the base of the tooth.

7.3 SPUR GEAR - LEWIS EQUATION FOR TOOTH BENDING STRESS

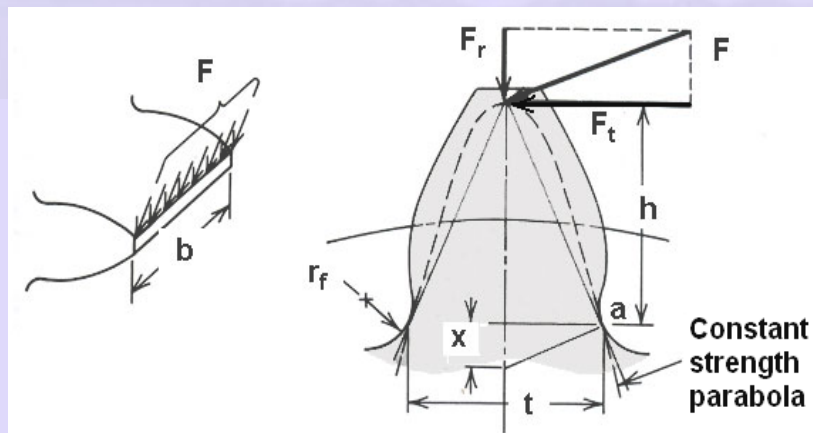


Fig. 7.3 Gear tooth as cantilever beam

Lewis considered gear tooth as a cantilever beam with static normal force F applied at the tip.

Assumptions made in the derivation are:

1. The full load is applied to the tip of a single tooth in static condition.
2. The radial component is negligible.
3. The load is distributed uniformly across the full face width.
4. Forces due to tooth sliding friction are negligible.
5. Stress concentration in the tooth fillet is negligible.

The Fig. 7.3 shows clearly that the gear tooth is stronger through out than the inscribed constant strength parabola, except for the section at 'a' where parabola and tooth profile are tangential to each other.

At point 'a', bending stress is

$$\sigma = \frac{Mc}{I} = \frac{6F_t h}{bt^2} \quad (7.6)$$

By similar triangles,

$$\frac{t/2}{x} = \frac{h}{t/2} \text{ or } \frac{t^2}{h} = 4x \quad (7.7)$$

Substituting of Eqn. (7.7) in Eqn. (7.6), it gives

$$\sigma = \frac{6F_t}{4bx} \quad (7.8)$$

$$y = \frac{2x}{3p} \quad (7.9)$$

where 'y' is defined as the Lewis form factor .

And substituting Eqn. (7.9) in Eqn. (7.8) we get

$$\sigma = \frac{F_t}{b p y} \quad (7.10)$$

Eqn. 7.10 is the basic Lewis equation in terms of circular pitch.

In SI units gears are more often made to standard modules. Hence by substituting

$p = \pi m$ in equation (7.10), we get

$$\sigma = \frac{F_t}{b \pi y m} \quad (7.11)$$

Let $Y = \pi y$, which is known as modified Lewis form factor, then

$$\sigma = \frac{F_t}{b Y m} \quad (7.12)$$

Eqn. 12 is the standard Lewis equation for tooth bending stress based on module.

Both Y and y are functions of tooth shape (but not size) and therefore vary with the number of teeth in the gear. These values can be obtained from Table 7.1 or Graph in Fig. 7.4.

Table 7.1 VALUES OF LEWIS FORM FACTOR

Number of teeth	$\phi = 20^\circ$ $a = 0.8m^*$ $b = m$	$\phi = 20^\circ$ $a = m$ $b = 1.25m$	$\phi = 25^\circ$ $a = m$ $b = 1.25m$	$\phi = 25^\circ$ $a = m$ $b = 1.35m^+$
12	0.335 12	0.229 60	0.276 77	0.254 73
13	0.348 27	0.243 17	0.292 81	0.271 77
14	0.359 85	0.255 30	0.307 17	0.287 11
15	0.370 13	0.266 22	0.320 09	0.301 00
16	0.379 31	0.276 10	0.331 78	0.133 63
17	0.387 57	0.285 08	0.342 40	0.325 17
18	0.395 02	0.293 27	0.352 10	0.335 74
19	0.401 79	0.300 78	0.360 99	0.345 46
20	0.407 97	0.307 69	0.369 16	0.354 44
21	0.413 63	0.314 06	0.376 71	0.362 76
22	0.418 83	0.319 97	0.383 70	0.370 48
24	0.428 06	0.330 56	0.396 24	0.384 39
26	0.436 01	0.339 79	0.407 17	0.396 57
28	0.442 94	0.347 90	0.416 78	0.407 33
30	0.449 02	0.355 10	0.425 30	0.416 91
34	0.459 20	0.367 31	0.439 76	0.433 23
38	0.467 40	0.377 27	0.451 56	0.446 63
45	0.478 46	0.390 93	0.467 74	0.465 11
50	0.484 58	0.398 60	0.476 81	0.475 55
60	0.493 91	0.410 47	0.490 86	0.491 77
75	0.503 45	0.422 83	0.505 46	0.508 77
100	0.513 21	0.435 74	0.520 71	0.526 65
150	0.523 21	0.449 30	0.536 68	0.545 56
300	0.533 48	0.463 64	0.553 51	0.565 70
Rack	0.544 06	0.478 97	0.571 39	0.587 39

* Stub teeth + Large fillet

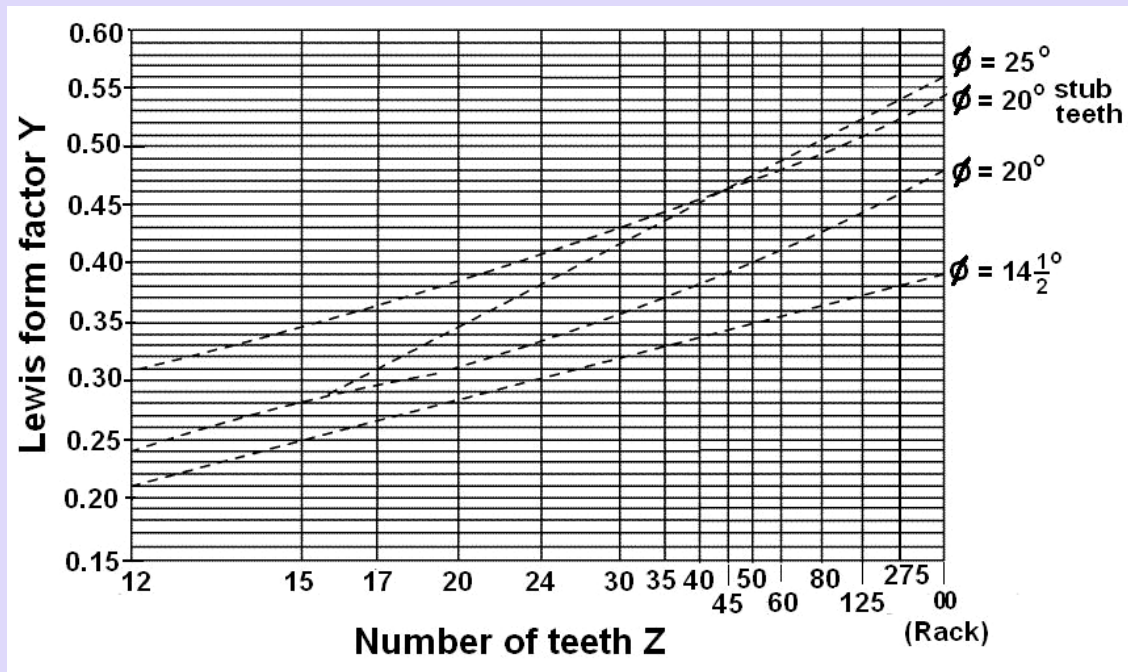


Fig. 7.4 spur gear - graph 1 for modified Lewis form factor

The Lewis equation indicates that tooth bending stress varies with the following:

$$\sigma = \frac{F_t}{b Y m} \quad (7.12)$$

- (1) Directly with load,
- (2) Inversely with tooth width b ,
- (3) Inversely with tooth size p or m ,
- (4) Inversely with tooth shape factor y or Y .

Drawbacks of Lewis equation are:

1. The tooth load in practice is not static. It is dynamic and is influenced by pitch line velocity.
2. The whole load is carried by single tooth is not correct. Normally load is shared by teeth since contact ratio is near to 1.5.
3. The greatest force exerted at the tip of the tooth is not true as the load is shared by teeth. It is exerted much below the tip when single pair contact occurs.
4. The stress concentration effect at the fillet is not considered.

SPUR GEAR – MODIFIED LEWIS EQUATION FOR BENDING STRESS

The modified Lewis equation for bending stress is,

$$\sigma = \frac{F_t}{K'_v b Y m} \quad (7.13)$$

where K'_v is known as velocity factor and is given by Barth's equation below for known pitch line velocity V in m/s and is given by,

$$K'_v = \frac{6}{6 + V} \quad (7.14)$$

Eqn. (7.14) is used for cut or milled teeth or for gears not carefully generated.

$$K'_v = \frac{50}{50 + (200V)^{0.5}} \quad (7.15)$$

Eqn. (7.15) is used for hobbed and shaped gears.

$$K'_v = \left[\frac{78}{78 + (200V)^{0.5}} \right]^{0.5} \quad (7.16)$$

Eqn. (7.16) is used for high-precision shaved or ground teeth.

The modified Lewis equation given in eqn. 7.13 is used when fatigue failure of the gear teeth is not a problem and a quick estimate is desired for more detailed analysis.

SPUR GEAR - TOOTH BENDING STRESS

Factors that influence gear tooth bending stresses are as follows:

1. Pitch line velocity.
2. Manufacturing accuracy.
3. Contact ratio.
4. Stress concentration.
5. Degree of shock loading.
6. Accuracy and rigidity of mounting.
7. Moment of inertia of the gears and attached rotating Members.

7.4 SPUR GEAR –TOOTH BENDING STRESS (AGMA)

Accommodating the earlier mentioned factors, American Gear Manufacturing Association (AGMA) came up with a refined form of Lewis equation as given below:

$$\sigma = \frac{F_t}{b m J} K_v K_o K_m \quad (7.17)$$

Where, J = Spur gear geometry factor. This factor includes the Lewis form factor Y and also a stress concentration factor based on a tooth fillet radius of $0.35/P$. It also depends on the number teeth in the mating gear.

$$J = \frac{Y}{K_f} \quad (7.18)$$

Where, Y is the modified Lewis form factor dealt earlier and K_f is the fatigue stress concentration factor given below:

$$K_f = H + \left(\frac{t}{r}\right)^L + \left(\frac{t}{l}\right)^M \quad (7.19)$$

Where, $H = 0.34 - 0.458 366 2\phi$ ----- (7.20)

$L = 0.316 - 0.458 366 2\phi$ ----- (7.21)

$M = 0.290 + 0.458 366 2\phi$ ----- (7.22)

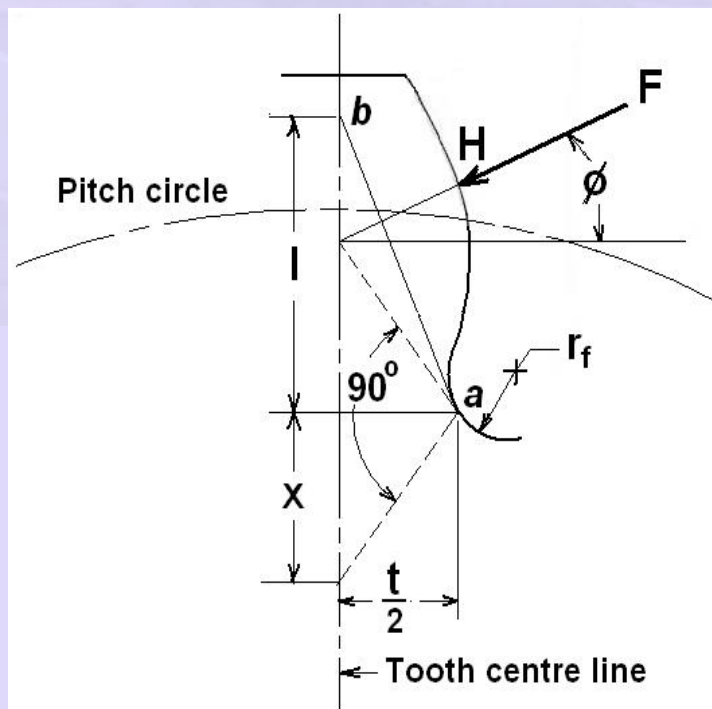


Fig. 7.4.Maximum Tooth loading

$$r = \frac{r_f + (b - r_f)^2}{(d/2) + b - r_f} \quad (7.23)$$

Where, r_f is the fillet radius, d is the pitch diameter and b is the dedendum.

J value can also be obtained from the Table 7.2 or Fig.7.5

Table 7.2 AGMA Geometry factor J for teeth having $\Phi = 20^\circ$, $a = 1m$, $b = 1.25m$ and $r_f = 0.3m$

Number of teeth	Number of teeth in mating gear							
	1	17	25	35	50	85	300	1000
18	0.244 86	0.324 04	0.332 14	0.338 40	0.344 04	0.350 50	0.355 94	0.361 12
19	0.247 94	0.330 29	0.338 78	0.345 37	0.351 34	0.358 22	0.364 05	0.369 63
20	0.250 72	0.336 00	0.344 85	0.351 76	0.358 04	0.365 32	0.371 51	0.377 49
21	0.253 23	0.341 24	0.350 44	0.357 64	0.364 22	0.371 86	0.378 41	0.384 75
22	0.255 52	0.346 07	0.355 59	0.363 06	0.369 92	0.377 92	0.384 79	0.391 48
24	0.259 51	0.354 68	0.364 77	0.372 75	0.380 12	0.388 77	0.396 26	0.403 60
26	0.262 89	0.362 11	0.372 72	0.381 15	0.388 97	0.398 21	0.406 25	0.414 18
28	0.265 80	0.368 60	0.379 67	0.388 51	0.396 73	0.406 50	0.415 04	0.423 51
30	0.268 31	0.374 62	0.385 80	0.395 00	0.403 59	0.413 83	0.422 83	0.431 79
34	0.272 47	0.383 94	0.396 71	0.405 94	0.415 17	0.426 24	0.436 04	0.445 86
38	0.275 75	0.391 70	0.404 46	0.414 80	0.424 56	0.436 33	0.446 80	0.457 35
45	0.280 13	0.402 23	0.415 79	0.426 85	0.437 35	0.450 10	0.461 52	0.473 10
50	0.282 52	0.408 08	0.422 08	0.435 55	0.444 48	0.457 78	0.469 75	0.481 93
60	0.286 13	0.417 02	0.431 73	0.443 83	0.455 42	0.469 60	0.482 43	0.495 57
75	0.289 79	0.426 20	0.441 63	0.454 40	0.466 68	0.481 79	0.495 54	0.509 70
100	0.293 53	0.435 61	0.451 80	0.465 27	0.478 27	0.494 37	0.509 09	0.524 35
150	0.297 38	0.445 30	0.462 26	0.476 45	0.490 23	0.507 36	0.523 12	0.539 54
300	0.301 41	0.455 26	0.473 04	0.487 98	0.502 56	0.520 78	0.537 65	0.555 33
Rack	0.305 71	0.465 54	0.484 15	0.499 88	0.515 29	0.534 67	0.552 72	0.571 73

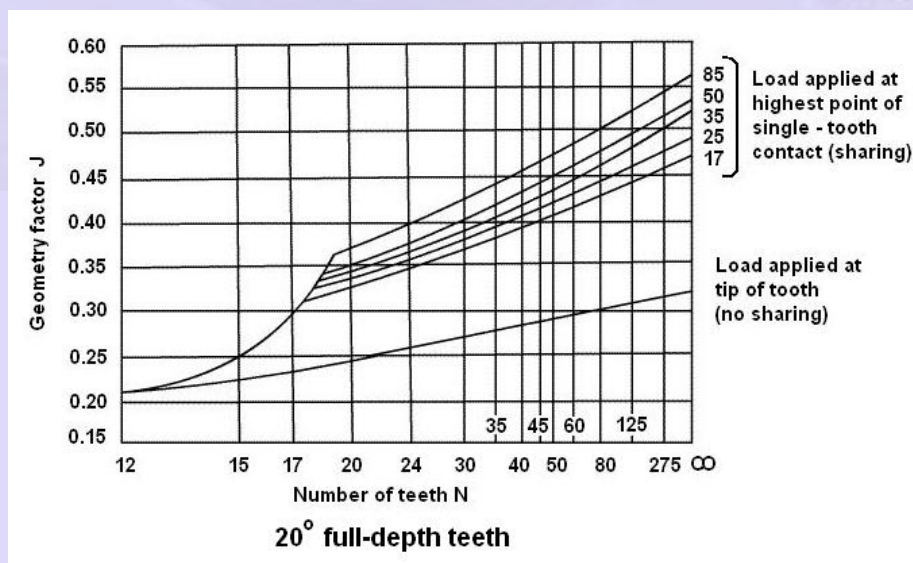


Fig. 7.5 Graph 2 AGMA geometry (J) factor

K_v - Velocity factor given by eqns. (7.24) to (7.26)

K_o = overload factor, given in Table 7.3

K_m = Load distribution factor, given in Table 7.4

K_v = Velocity or dynamic factor, indicates the severity of impact as successive pairs of teeth engage. This is a function of pitch line velocity and manufacturing accuracy. It is given by inverse of Barth's equation (7.24), (7.25) & (7.26) or very rough value from the Fig.7 6.

$$K_v = \frac{6+V}{6} \quad (7.24)$$

Eqn. (7.24) is used for cut or milled teeth or for gears not carefully generated.

$$K_v = \frac{50 + (200V)^{0.5}}{50} \quad (7.25)$$

Eqn. (7.25) is used for hobbed and shaped gears.

$$K_v = \left[\frac{78 + (200V)^{0.5}}{78} \right]^{0.5} \quad (7.26)$$

Eqn. (7.26) is used for high-precision shaved or ground teeth.

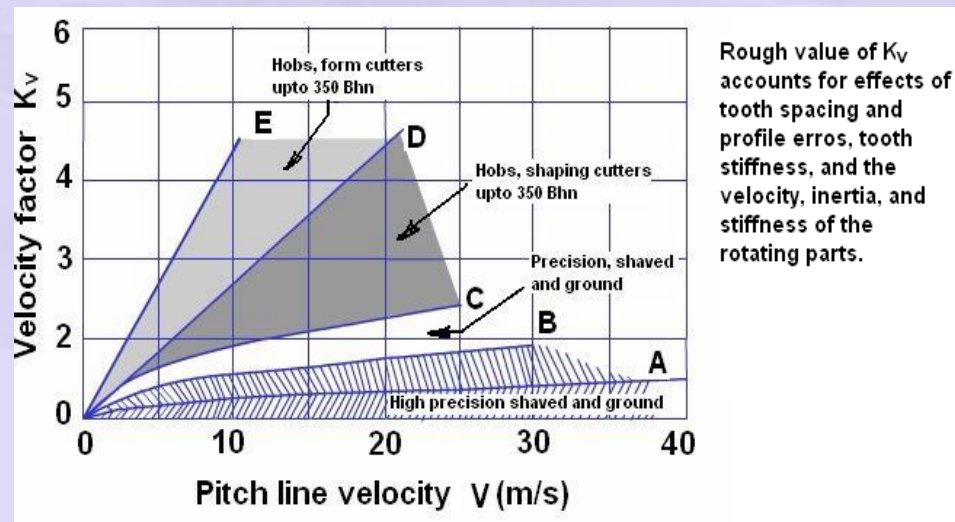


Fig. 7.6 Velocity factor K_v

K_o = Overload factor which reflects the degree of non-uniformity of driving and load torques. It is given in Table 7.3

K_m = Load distribution factor which accounts for non-uniform spread of the load across the face width. It depends on the accuracy of mounting, bearings, shaft deflection and accuracy of gears.

Table 7. 3 -Overload factor K_o

	Driven Machinery		
Source of power	Uniform	Moderate Shock	Heavy Shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Table 7.4 Load distribution factor K_m

	Face width (mm)			
Characteristics of Support	0 - 50	150	225	400 up
Accurate mountings, small bearing clearances, minimum deflection, precision gears	1.3	1.4	1.5	1.8
Less rigid mountings, less accurate gears, contact across the full face	1.6	1.7	1.8	2.2
Accuracy and mounting such that less than full-face contact exists	Over 2.2	Over 2.2	Over 2.2	Over 2.2

7.5 SPUR GEAR – PERMISSIBLE TOOTH BENDING STRESS (AGMA)

Endurance limit of the material is given by:

$$\sigma_e = \sigma_e' k_L k_v k_s k_r k_T k_f k_m \quad (7.27)$$

Where, σ_e' endurance limit of rotating-beam specimen

k_L = load factor , = 1.0 for bending loads

k_v = size factor, = 1.0 for $m < 5$ mm and
= 0.85 for $m > 5$ mm

k_s = surface factor, is taken from Fig. 7.7 based on the ultimate tensile strength of the material
for cut, shaved, and ground gears.

k_r = reliability factor given in Table 7.5.

k_T = temperature factor, = 1 for $T \leq 350^\circ\text{C}$
= 0.5 for $350 < T \leq 500^\circ\text{C}$

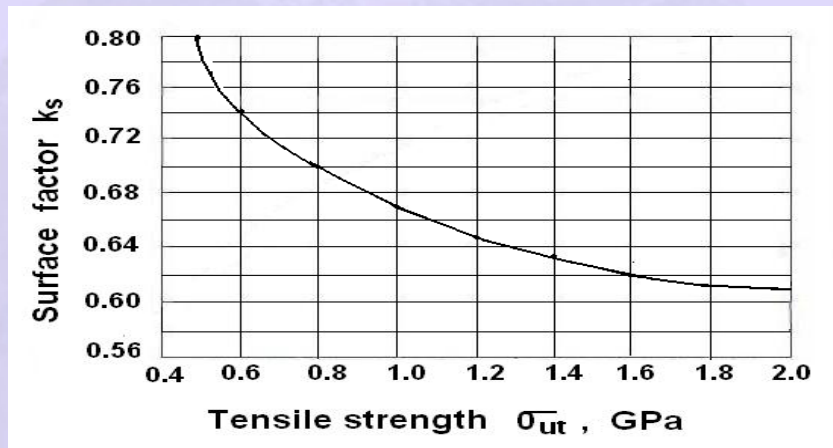


Fig. 7.7 Surface factor k_s

Table 7.5 Reliability Factor k_r

Reliability R	0.50	0.90	0.95	0.99	0.999	0.9999
Reliability Factor k_r	1.000	0.897	0.868	0.814	0.753	0.702

k_f = fatigue stress concentration factor. Since this factor is included in J factor, its value is taken as 1.

k_m = Factor for miscellaneous effects. For idler gears subjected to two way bending, = 1. For other gears subjected to one way bending, the value is taken from the Fig.7.8. Use $k_m = 1.33$ for σ_{ut} less than 1.4 GPa.

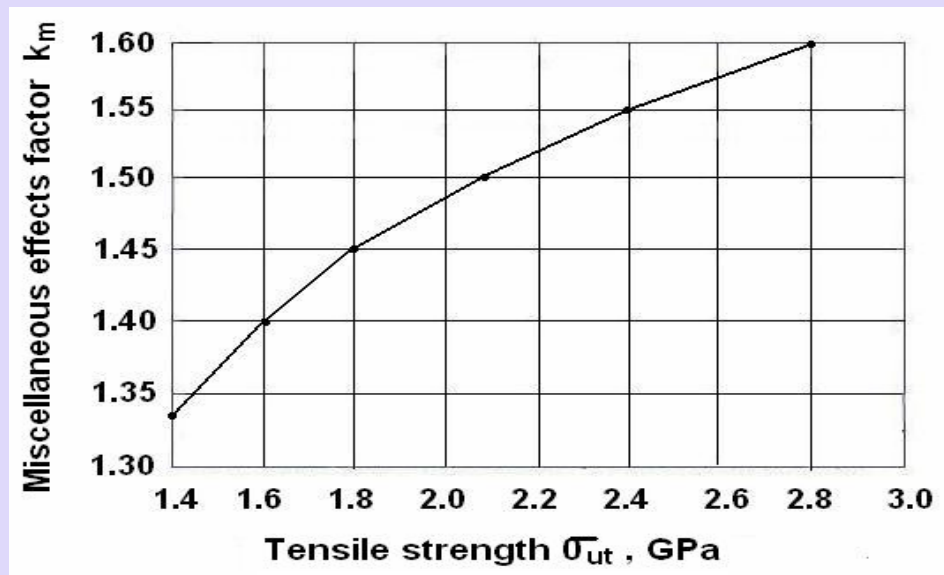


Fig. 7.8 - Miscellaneous effects factor k_m

7.6 Permissible bending stress

$$[\sigma] = \frac{\sigma_e}{s} \quad (7.28)$$

where s is the factor of safety.

Hence the design equation from bending consideration is :

$$\sigma \leq [\sigma] \quad (7.29)$$

7.7 SPUR GEARS – BUCKINGHAM'S DYNAMIC LOAD EQUATION

Buckingham's dynamic load equation (1932):

According to him, small machining error and deflection of teeth under load cause periods of acceleration, inertia forces, and impact loads on the teeth similar to variable load superimposed on a steady load. The total maximum instantaneous load on the teeth or dynamic load is F_d

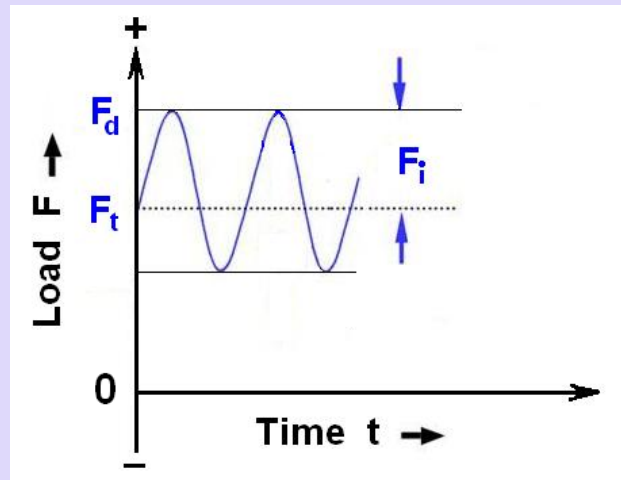


Fig. 7.9 Dynamic loading on gear

$$F_d = F_t + F_i \quad (7.30)$$

$$F_i = \left[\frac{9.84V(Cb + F_t)}{9.84V + 0.4696\sqrt{Cb + F_t}} \right] \quad (7.31)$$

Where

F_d – dynamic load, N

F_t – transmitted load, N

F_i – increment load due to machining errors and the pitch line velocity

V – pitch line velocity, m/s

b – face width, mm

C – factor depending on machining error

Table 7.6 Value of C for spur gears

Tooth Form	Material of Pinion and gear	C
14.5°	Cast iron and Cast iron	5720 e
	Steel and Cast iron	7850 e
	Steel and Steel	11440 e
20° Full depth	Cast iron and Cast iron	5930 e
	Steel and Cast iron	8150 e
	Steel and Steel	11860 e
20° stub tooth	Cast iron and Cast iron	6150 e
	Steel and Cast iron	8450 e
	Steel and Steel	12300 e

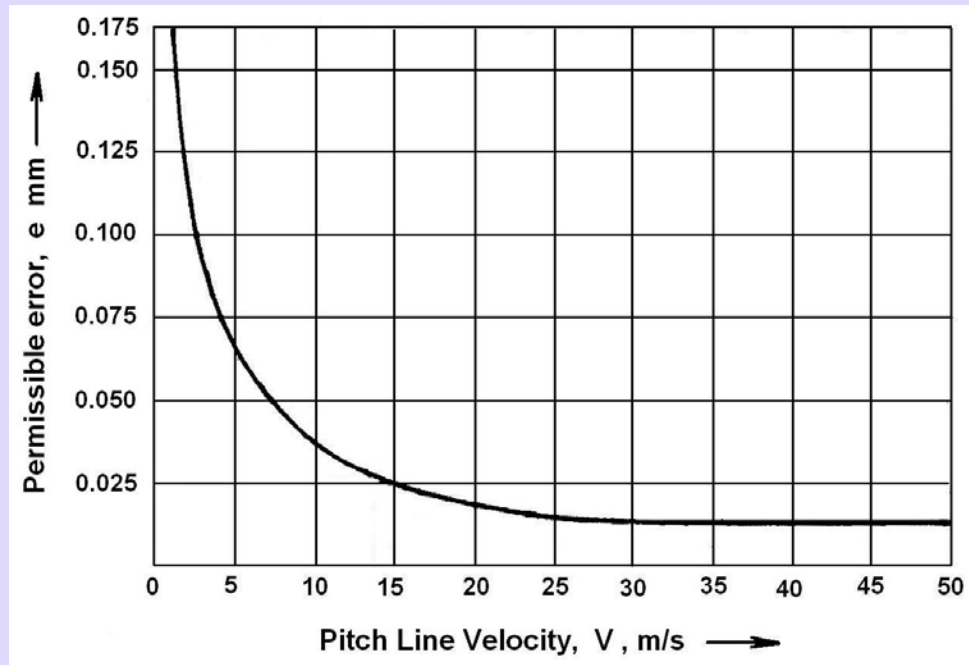


Fig. 7.10 Graph 6 Permissible error

Table 7.7 Expected error in tooth profile

Module	Gear quality and expected error e		
	First class Commerical gears	Carefully cut gears	Precision gears
upto 4	0.050	0.025	0.0125
5	0.056	0.025	0.0125
6	0.064	0.030	0.0150
7	0.072	0.035	0.0170
8	0.080	0.038	0.0190
9	0.085	0.041	0.0205
10	0.090	0.044	0.0220

SPUR GEARS –BUCKINGHAM’S DYNAMIC LOAD FOR DESIGN

Lewis equation is,

$$\sigma = \frac{F_t}{b Y m} \quad (7.12)$$

Rearranging eqn 12,

$$F_t = \sigma b Y m \quad (7.32)$$

If we substitute permissible stress in Eqn. (7.32) we get on the right side, beam or tooth strength of the gear F_{td} as,

$$F_{td} = [\sigma] b Y m \quad (7.33)$$

From design point of view $F_{td} \geq F_d$

