

**Microfluidics (ME60310)/ Micro-scale Fluid Flow and Heat Transfer
(ME41616), End-Semester Examination, April 2015, IIT Kharagpur,**

Full Marks = 100

All questions are compulsory, carrying equal marks.

Q1.

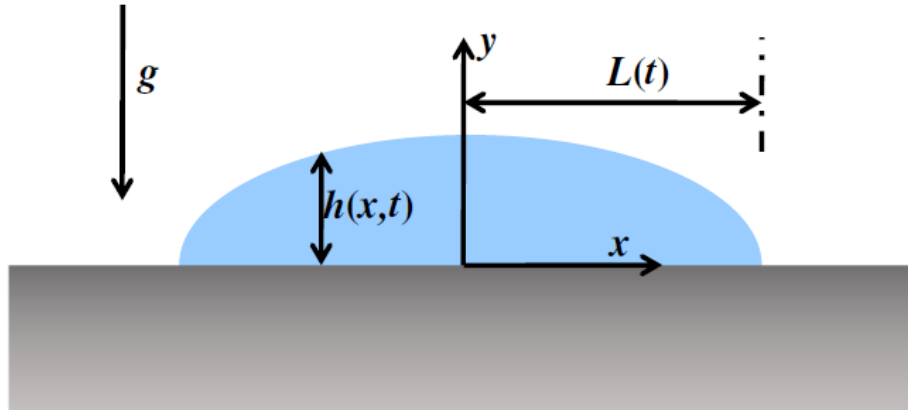
- (a) An oscillatory pressure gradient (of the form: $-\frac{1}{\rho} \frac{dp}{dx} = -\frac{1}{\rho} \frac{dp}{dx}\bigg|_0 + A \sin \omega t$) is applied on a fluid flowing through a parallel plate microchannel, with no slip at the walls. Determine the velocity field as a function of position and time.
- (b) What are the major limitations of the Stokes law for calculating viscous drag, even in low Reynolds number limits?

Q2. A parallel plate horizontal micro-capillary is being filled up with blood, which is effectively treated as a non-Newtonian fluid, the stress tensor for which is described as:

$\boldsymbol{\tau} = \mu_{app} [2\boldsymbol{\Gamma}]$, where $\boldsymbol{\Gamma} = \frac{1}{2} [\nabla \mathbf{v} + \nabla \mathbf{v}^T]$ is the rate of strain tensor. For simplicity, the constitutive behaviour of blood is approximately described by a power law model, described by $\mu_{app} = a(2\boldsymbol{\Gamma})^{n-1}$, where $\boldsymbol{\Gamma} = \left[\frac{1}{2} (\boldsymbol{\Gamma} : \boldsymbol{\Gamma}) \right]^{\frac{1}{2}}$. In effect, this reduces to $\boldsymbol{\tau} = a\dot{\boldsymbol{\gamma}}^n$,

where $\dot{\boldsymbol{\gamma}}$ is the rate of deformation. The viscous resistances against the capillary movement are approximated from fully developed flow considerations. Assuming a constant contact angle, derive a governing differential equation for capillary meniscus displacement, as a function of time, by using a reduced order model. There is no need to solve this equation.

Q3. Consider a long cylindrical drop spreading on a flat horizontal surface due to gravitational forces acting on it (as shown in the figure). Determine the height of the drop (h) as a function of t and x . Also find the radius of the drop $L(t)$ as a function of time. State the assumptions that you make.



Q4.

Consider pressure-driven flow of a 1:1 symmetric electrolyte in a parallel plate microchannel. The zeta potential is small enough so that the Debye-Hückel linearization is valid. Derive an expression for the streaming potential induced in its most simplified form, and express the same through a suitable non-dimensional scheme.

Q5. Consider the advection-diffusion of a species in a parallel plate microchannel of height $2H$. Following Taylor's approach, derive an expression for the effective dispersion coefficient as function of the Peclet number and the channel height. State all the assumptions that you make. The velocity profile can be taken as:

$$\frac{u}{u_{av}} = \frac{3}{2} \left(1 - \frac{y^2}{H^2} \right),$$

where y is the transverse coordinate.