Integral Forms of Control Volume Conservation Equations (Reynolds Transport theorem) (Lectures 21 to 27)

Q1. **Choose the correct answer**

(i) Mathematical statement of Reynolds transport theorem is given by

(a) $\left.\frac{dN}{dt}\right|_{\text{system}} = \frac{\partial}{\partial t}\int_{CS} \eta \vec{V}_r .\, d\vec{A} + \int_{C\forall} \eta\rho d\forall$

(b) $\left.\frac{dN}{dt}\right|_{\text{system}} = \frac{\partial}{\partial t}\int_{CS} \eta\rho \vec{V}_r . d\vec{A} + \int_{C\forall} \eta\rho d\forall$

(c) $\left.\frac{dN}{dt}\right|_{\text{system}} = \frac{\partial}{\partial t}\int_{CV} \eta\rho d\forall - \int_{CS} \eta\left(\rho \vec{V}_r . \hat{n}\right) dA$

(d) $\left.\frac{dN}{dt}\right|_{\text{system}} = \frac{\partial}{\partial t}\int_{CV} \eta\rho d\forall + \int_{CS} \eta\left(\rho \vec{V}_r . \hat{n}\right) dA$

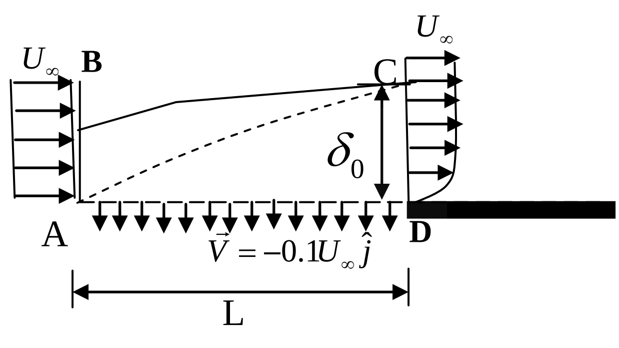
where N is an extensive property, η is the property per unit mass, $\vec{V}_r$ is the velocity of fluid relative to the control volume and $d\vec{A}$ is the elemental area vector on the control surface.

[*Ans.(*d)]

Q2.

A fluid of constant density ρ flows steadily past a porous plate with a uniform free stream velocity U_∞ as shown in the figure. Constant suction is applied along the porous section. The velocity distribution at section CD is given by $\frac{u}{U_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$.

Determine the mass flow rate per unit width of the plate perpendicular to the plane of the figure across the section BC.



Solution

From conservation of mass for the CV, we get

$$0 = 0 + \rho\int_0^\delta u dy w + \dot{m}_{BC} + 0.1\rho U_\infty w L - \rho U_\infty w\delta$$

or $$\dot{m}_{BC} = \rho U_\infty w\delta - \rho\int_0^\delta u dy w - 0.1\rho U_\infty wL$$

or $$\dot{m}_{BC} = \rho U_\infty w\delta - \rho U_\infty w\int_0^\delta \left[\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right]dy - 0.1\rho U_\infty wL$$

or $$\dot{m}_{BC} = \rho U_\infty w\delta - \rho U_\infty w\left(\frac{3\delta}{4} - \frac{\delta}{8}\right) - 0.1\rho U_\infty wL$$

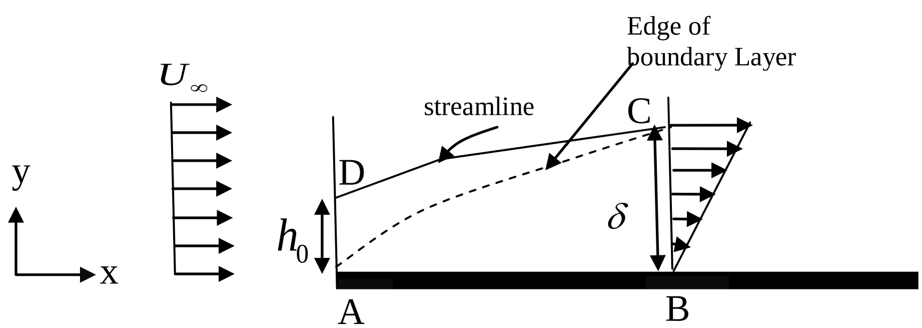
or $$\dot{m}_{BC} = \rho U_\infty w\delta - \rho U_\infty w\frac{5\delta}{8} - 0.1\rho U_\infty wL$$

or $$\dot{m}_{BC} = \rho U_\infty w\frac{3\delta}{8} - 0.1\rho U_\infty wL$$

The mass flow rate per unit width of the plate perpendicular to the plane of the figure across the section BC is $\rho U_\infty\left(\frac{3\delta}{8} - 0.1L\right)$

Q3.

A free stream of uniform velocity U_∞ interacts with a flat plate of length, and width b perpendicular to the plane of the figure. Because of viscous interaction, a boundary layer tends to develop. The velocity profile within the boundary layer may be assumed to be approximately linear. Consider a streamline D-C that passes through the edge of the boundary layer at $y = \delta$ (i.e., the point 2). For $\delta = 1$ mm, $L = 1$ m, $b = 1$ m, ρ(density of fluid) $= 1000$ kg/m^3 and $U_\infty = 1$ m/s, find the magnitude of the net drag force acting on the plate.



Solution

From conservation of mass for the control volume ABCD,

$$0 = 0 + \int_{AD}\rho(\vec{V}.\hat{n})dA + \int_{BC}\rho(\vec{V}.\hat{n})dA$$

or $$0 = 0 + \rho\int_0^{h_0}(-U_\infty)bdy + \int_0^\delta ubdy$$

or $$U_\infty h_0 = \int_0^\delta u dy$$

or $$U_\infty h_0 = \int_0^\delta U_\infty \frac{y}{\delta} dy$$

or $$U_\infty h_0 = \frac{U_\infty \delta}{2}$$

or $$h_0 = \frac{\delta}{2} = \frac{1}{2} = 0.5 \text{ mm}$$

Applying the momentum conservation equation for the control volume ABCD, we have

$$F = \int_{AD} \rho u(\vec{V}.\hat{n})dA + \int_{BC} \rho u(\vec{V}.\hat{n})dA$$

$$= \rho \int_0^{h_0} U_\infty \left(-U_\infty\right) bdy + \rho \int_0^\delta u\left(u\right) bdy$$

$$= -\rho U_\infty^2 b h_0 + \rho b \int_0^\delta u^2 dy$$

$$= -\rho U_\infty b \int_0^\delta u dy + \rho b \int_0^\delta u^2 dy$$

$$= \rho b \left[\int_0^\delta u^2 dy - U_\infty \int_0^\delta u dy \right]$$

$$= \rho b \left[\int_0^\delta U_\infty^2 \frac{y^2}{\delta^2} dy - U_\infty \int_0^\delta U_\infty \frac{y}{\delta} dy \right]$$

$$= \rho b U_\infty^2 \left[\frac{\delta^3}{3\delta^2} - \frac{\delta^2}{2\delta} \right]$$

$$= -\frac{\rho b U_\infty^2 \delta}{6}$$

Substituting the respective values, we get

$$\left|F\right| = \frac{\rho b U_\infty^2 \delta}{6} = \frac{1000 \times 1 \times \left(1\right)^2 \times 1 \times 10^{-3}}{6} = 0.167 \text{ N}$$