Introduction and Fundamental Concepts (Lectures 1-7)

Q1. **Choose the correct answer**

(i) A fluid is a substance that

(a) has the same shear stress at a point regardless of its motion
(b) is practically incompressible
(c) cannot remain at rest under action of any shear force
(d) obeys Newton's law of viscosity

[*Ans.(*c)]

(ii) For a Newtonian fluid

(a) shear stress is proportional to shear strain
(b) rate of shear stress is proportional to shear strain
(c) shear stress is proportional to rate of shear strain
(d) rate of shear stress is proportional to rate of shear strain

[*Ans.(*c)]

(iii) Shear stress for a general fluid motion is represented by $\tau = \mu \left(\frac{du}{dy} \right)^n + A$, where n and A are constants. A Newtonian fluid is given by

(a) $n > 1$ and $A = 0$
(b) $n = 1$ and $A = 0$
(c) $n > 1$ and $A \neq 0$
(d) $n < 1$ and $A = 0$

[*Ans.(*b)]

(iv) If the relationship between the shear stress τ and the rate of shear strain $\frac{du}{dy}$ is expressed as $\tau = m\left[\frac{du}{dy}\right]^n$. The fluid with the exponent $n < 1$ is known as

(a) Pseudoplastic fluid
(b) Bingham fluid
(c) Dilatant fluid
(d) Newtonian fluid

[*Ans.(*a)]

(v) The increase in temperature

(a) increases the viscosity of a liquid and decreases the viscosity of a gas
(b) decreases the viscosity of a liquid and increases the viscosity of a gas
(c) increases the viscosity of both a liquid and a gas
(d) decreases the viscosity of both a liquid and a gas

[*Ans.(*b)]

(vi) The bulk modulus of elasticity for an ideal gas (equation of state $p = \rho RT$, where p is the pressure, ρ is the density, R is the characteristic gas constant and T is the absolute temperature) at constant temperature T is given by

(a) $\frac{p}{\rho}$

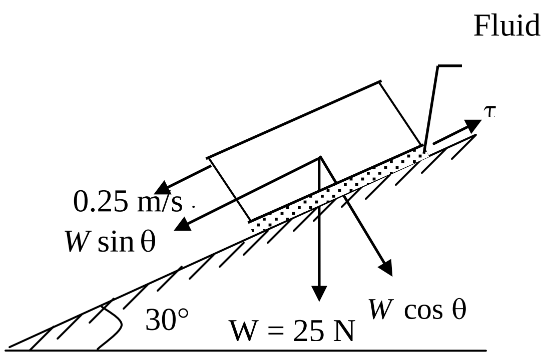
(b) RT
(c) p
(d) ρRT

[*Ans.*(c)]

Q2.
A plate having an area of 0.4 m^2 is sliding down the inclined plane at 30° to the horizontal with a velocity of 0.25 m/s. There is a cushion of fluid 2 mm thick between the plane and the plate. The weight of the plate is 25 N. Assuming linear velocity profile in the film, find the viscosity of the fluid.

Solution

The arrangement is shown in the figure below.



Component of weight along the slope is $W \sin 30°$.
Velocity gradient is found to be

$$\frac{du}{dy} = \frac{V - 0}{h} = \frac{V}{h}$$

where h is the thickness of the oil film and V is the velocity of the plate. .
Viscous resistance F is given by

$$F = \tau A$$

or $$F = \mu \frac{du}{dy} A = \mu \frac{V}{h} A$$

At equilibrium, the viscous resistance to the motion should be equal to the component of the weight of the solid block along the slope. Thus,

$$\mu \frac{V}{h} A = W \sin 30°$$

or $$\mu \times \frac{0.25}{2 \times 10^{-3}} \times 0.4 = 25 \sin 30°$$

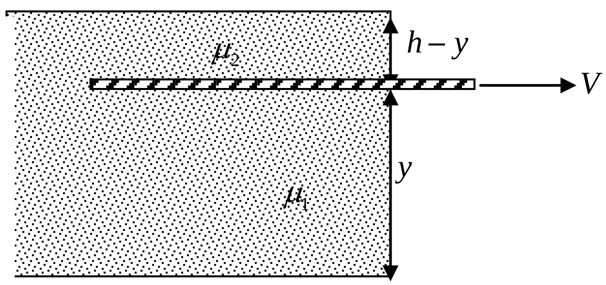
or $$\mu = 0.25 \text{ N-s/m}^2$$

Q3.
A thin plate is placed between two flat surfaces h apart such that the viscosity of liquids on the top and bottom of the plate are μ_1 and μ_2 respectively. Determine the position of the plate such that the viscous resistance to uniform motion of the plate is minimum.

Solution

Let us assume that the velocity of the plate be V .

Let F_1 and F_2 be the shear forces per unit area on the lower surface and upper surface of the thin plate respectively. Let us also consider that the distance of the thin plate from the bottom wall is y as shown in the figure below.



From Newton's law of viscosity, shear stress on the bottom surface of the plate τ_1 is given by

$$\tau_1 = \mu_1 \frac{du}{dy}$$
$$= \mu_1 \frac{V}{y}$$

Shear force per unit area on the bottom surface of the plate is

$$F_1 = \mu_1 \frac{du}{dy} = \mu_1 \frac{V}{y}$$

From Newton's law of viscosity, shear stress on the upper surface of the plate τ_2 is given by

$$\tau_2 = \mu_2 \frac{du}{dy}$$

where $dy = h - y$ (Neglecting thickness of the plate)

or $$\tau_2 = \mu_2 \frac{V}{h-y}$$

Shear force per unit area on the upper surface of the plate is

$$F_2 = \mu_2 \frac{du}{dy} = \mu_2 \frac{V}{h-y}$$

Total viscous resistance to drag the plate is

$$F = F_1 + F_2$$
$$= \mu_1 \frac{V}{y} + \mu_2 \frac{V}{h-y}$$

For minimum F, we have

$$\frac{dF}{dy}=0$$

or
$$-\mu_1\frac{V}{y^2}+\mu_2\frac{V}{(h-y)^2}=0$$

or
$$\mu_2\frac{1}{(h-y)^2}=\mu_1\frac{1}{y^2}$$

or
$$\frac{y^2}{(h-y)^2}=\frac{\mu_1}{\mu_2}$$

or
$$\frac{y}{(h-y)}=\sqrt{\frac{\mu_1}{\mu_2}}$$

or
$$y=\frac{h\sqrt{\mu_1/\mu_2}}{1+\sqrt{\mu_1/\mu_2}}$$

Q4.

A uniform film of oil 0.13 mm thick separates two discs, each of 200 mm diameter, mounted coaxially. Ignoring the edge effects, calculate the torque necessary to rotate one disc relative to other at a speed of 7 rev/s, if the oil has a viscosity of 0.14 Pa-s.

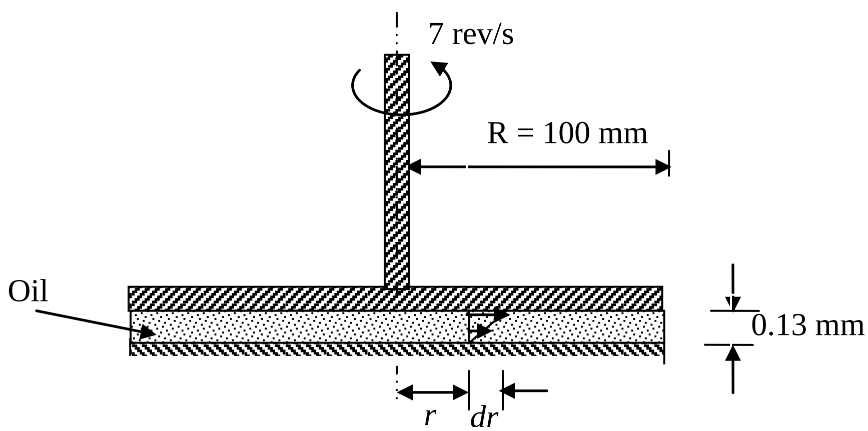
Solution

At a radial distance r (measured from the axis of the discs) in the oil film (figure below), the velocity

$$v=2\pi\times7\times r$$

$$\frac{dv}{dy}=\frac{2\pi\times7\times r}{0.13\times10^{-3}}=3.38\times10^{5}r$$

(y is measured along the direction of the axis i.e., perpendicular to the discs)



The force acting on an elemental portion of the disc of thickness dr at a radial location r is given by

$$dF=0.14\times\left(3.38\times10^{5}r\right)\times2\pi rdr$$

Corresponding torque

$$dT = 0.14 \times \left(3.38 \times 10^5 r\right) \times 2\pi r dr \times r$$

Hence,

$$T = \int dT = \int_0^R 0.14\left(3.38 \times 10^5 r\right) 2\pi r^2 dr$$

(where R, the radius of the disc = 100 mm = 0.1 m)

$$= \frac{0.14 \times 3.38 \times 10^5 \times 2\pi\left(0.1\right)^4}{4} = 7.43 \text{ N-m}$$

Q5.

(a) Find the change in volume of 1.00 m^3 of water at 26.7°C when subjected to a pressure increase of 2 MN/m^2 (The bulk modulus of elasticity of water at 26.7°C is 2.24×10^9 N/m^2).

(b) From the following test data, determine the bulk modulus of elasticity of water: at 3.5 MN/m^2 , the volume was 1.00 m^3 and at 24 MN/m^2 , the volume was 0.99 m^3.

Solution

(a)

Change in pressure $\Delta p = 2 \times 10^6$ N/m^2

Initial volume of water $\cancel{V} = 1.00$ m^3

Bulk modulus of elasticity is given by

$$E = \frac{\Delta p}{\dfrac{\Delta \forall}{\forall}}$$

or

$$\Delta \forall = \Delta p \frac{\forall}{E}$$

$$= 2 \times 10^6 \times \frac{1.00}{2.24 \times 10^9} = 0.893 \times 10^{-3} \text{ m}^3$$

(b) Change in pressure $\Delta p = \left(24 - 3.5\right) = 20.5 \text{MN/m}^2 = 20.5 \times 10^6$ N/m^2

Change in volume $\Delta \forall = \left(1 - 0.99\right) = 0.01$ m^3

The bulk modulus of elasticity of water is

$$E = \forall \frac{\Delta p}{\Delta \forall} = 1.00 \times \frac{20.5 \times 10^6}{0.01} = 2.05 \times 10^9 \text{ N/m}^2$$

Q6.

A spherical soap bubble of diameter d_1 coalesces with another bubble of diameter d_2 to form a single bubble of diameter d_3 containing the same amount of air. Assuming isothermal process, derive an analytical expression for d_3 as a function of d_1, d_2, the ambient pressure p_0 and the surface tension of soap solution σ. If $d_1 = 20$ mm, $d_2 = 40$ mm, $p_0 = 101$ kN/m^2 and $\sigma = 0.09$ N/m, determine d_3.

Solution

From conservation of mass

$$m_1 + m_2 = m_3$$

where m_1, m_2 and m_3 are the masses of air inside the bubbles of diameter d_1, d_2 and d_3 respectively.

For an isothermal process (considering air to behave as an ideal gas), the above leads to

$$p_1 d_1^3 + p_2 d_2^3 = p_3 d_3^3$$

where p_1, p_2 and p_3 are the pressures inside the bubbles of diameter d_1, d_2 and d_3 respectively.

Now,

$$p_1 = p_0 + \frac{8\sigma}{d_1}$$

$$p_2 = p_0 + \frac{8\sigma}{d_2}$$

$$p_3 = p_0 + \frac{8\sigma}{d_3}$$

Hence,

$$\left(p_0 + \frac{8\sigma}{d_1}\right) d_1^3 + \left(p_0 + \frac{8\sigma}{d_2}\right) d_2^3 = \left(p_0 + \frac{8\sigma}{d_3}\right) d_3^3$$

For the given values of $d_1 = 20$ mm, $d_2 = 40$ mm, $p_0 = 101$ kN/m^2, $\sigma = 0.09$ N/m

$$\left(101 + \frac{8 \times 0.09 \times 10^{-3}}{0.02}\right)(0.02)^3 + \left(101 + \frac{8 \times 0.09 \times 10^{-3}}{0.04}\right)(0.04)^3 = \left(101 + \frac{8 \times 0.09 \times 10^{-3}}{d_3}\right) d_3^3$$

or $\quad d_3^4 + 7.13 \times 10^{-6} d_3^3 = 72.3 \times 10^{-6} d_3$

which gives $\quad d_3 = 41.6 \times 10^{-3}$ m $= 41.6$ mm