Introduction to Fluid Machines (Lectures 49 to 53)

Q1. **Choose the correct answer**

(i) A hydraulic turbine rotates at N rpm operating under a net head H and having a discharge Q while developing an output power P. The specific speed is expressed as

(a) $\dfrac{N\sqrt{Q}}{(gH)^{3/4}}$

(b) $\dfrac{N\sqrt{P}}{\rho^{1/2}(gH)^{5/4}}$

(c) $\dfrac{N\sqrt{P}}{\rho^{1/2}(gH)^{3/4}}$

(d) $\dfrac{N\sqrt{Q}}{\rho^{1/2}(gH)^{5/4}}$

[*Ans.(*b)]

(ii) A hydraulic pump rotates at N rpm operating under a net head H and having a discharge Q . The specific speed is expressed as

(a) $\dfrac{N\sqrt{Q}}{(gH)^{3/4}}$

(b) $\dfrac{N\sqrt{P}}{\rho^{1/2}(gH)^{5/4}}$

(c) $\dfrac{N\sqrt{P}}{\rho^{1/2}(gH)^{3/4}}$

(d) $\dfrac{N\sqrt{Q}}{\rho^{1/2}(gH)^{5/4}}$

[*Ans.(*a)]

(iii) Two hydraulic turbines are similar and homologous when there are geometrically similar and have
(a) the same specific speed
(b) the same rotational speed
(c) the same power output
(d) the same Thoma's number

[*Ans.(*a)]

(iv) A turbomachine becomes more susceptible to cavitation if
(a) velocity attains a high value
(b) pressure falls below the vapour pressure
(c) temperature rises above a critical value
(d) Thomas cavitation parameter exceeds a certain limit

[*Ans.*(b)]

(v) Governing of turbines means

(a) the discharge is kept constant under all conditions

(b) allow the turbine to run at ‘ runaway’ speed

(c) the speed is kept constant under all conditions(loads)

(d) the power developed is kept constant under all conditions

[*Ans.*(c)]

Q2.

A Pelton wheel operates with a jet of 150 mm diameter under the head of 500 m. Its mean runner diameter is 2.25 m and it rotates with a speed of 375 rpm. The angle of bucket tip at outlet is 15°, coefficient of velocity is 0.98, mechanical losses equal to 3% of power supplied and the reduction in relative velocity of water while passing through bucket is 15%. Find (i) the force of jet on the bucket, (ii) the power developed (iii) bucket efficiency and (iv) the overall efficiency.

Solution

Inlet jet velocity is

$$V_1 = C_v\sqrt{2gH} = 0.98\times\sqrt{2\times 9.81\times 500} = 97.06 \text{ m/s}$$

Flow rate of water $Q = \frac{\pi}{4} d^2 V_1$

$$= \frac{\pi}{4}\times\left(0.13\right)^2\times 97.06 = 1.715 \text{ m}^3\text{/s}$$

Bucket speed is

$$U = \frac{\pi DN}{60} = \frac{\pi\times 2.55\times 375}{60} = 44.18 \text{ m/s}$$

From inlet velocity diagram,

Velocity of jet relative to wheel at inlet

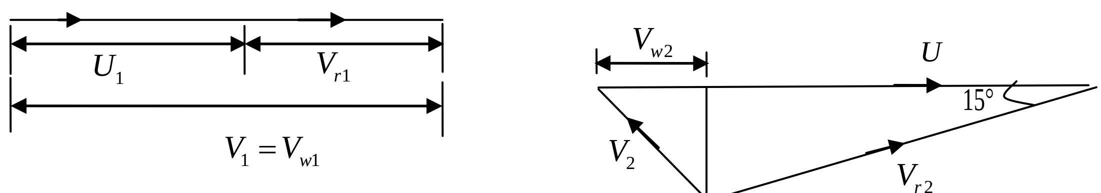
$$V_{r1} = V_1 - U = 97.06 - 44.8 = 52.88 \text{ m/s}$$

Tangential component of inlet jet velocity

$$V_{w1} = V_1 = 86.82 \text{ m/s}$$

It is given that

$$V_{r2} = 0.85V_{r1} = 0.85\times 52.88 = 44.95 \text{ m/s}$$



Inlet Outlet

From outlet velocity triangle as shown,

$$V_{w2} = U - V_{r2}\cos\beta_2$$
$$= 44.18 - 44.95\cos 15° = 0.76 \text{ m/s}$$

Force imparted on the bucket is

$$F = \rho Q\left(V_{w1} - V_{w2}\right)$$
$$= 103 \times 1.715\left(97.06 - 0.76\right)$$
$$= 165.15 \times 10^3 \text{ N} = 165.15 \text{ kN}$$

Power developed $P = FU$

$$= 165.15 \times 44.18 \text{ kW}$$
$$= 7.3 \times 10^3 \text{ kW} = 7.3 \text{ MW}$$

Bucket efficiency $\eta_b = \dfrac{2 \times 7.3 \times 10^3}{10^3 \times 1.715 \times \left(97.06\right)^2} = 0.903 \text{ or } 90.3\%$

Overall efficiency $\eta_o = \eta_b\left(1 - 0.03\right)$

$$= 0.903 \times 0.97 = 0.876 \text{ or } 87.6\%$$

Q3.

An inward flow reaction turbine working under a head of H has inlet and outlet vane angles both equal to 90°. Show that the peripheral velocity is given by

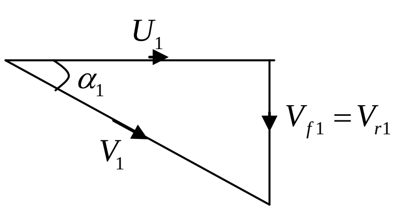
$$U = \sqrt{\frac{2gH}{2 + C^2 \tan^2_{\alpha_1}}}$$

where C is the ratio of velocity of flow at outlet and inlet and α_1 is the guide vane angle. Also show that the hydraulic efficiency can be expressed as

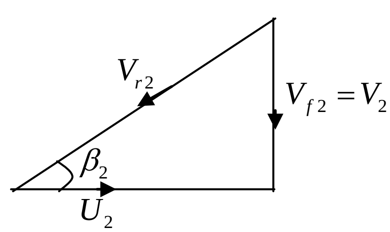
$$\eta_h = \frac{2}{2 + C^2 \tan^2 \alpha_1}$$

Solution

The inlet and outlet velocity triangles are shown in the figure below.



Inlet velocity triangle



Outlet velocity triangle

From the inlet velocity triangle, we have

$$V_{w1} = U_1$$
$$\tan \alpha_1 = \frac{V_{f1}}{V_{w1}}$$

or $V_{w1} = V_{f1} \cot \alpha_1$

When water flows through the vanes, we get

$$H - \frac{V_{f2}^{\;2}}{2g} = \frac{1}{g} V_{w1} U_1$$

or $$H = \frac{1}{g} V_{w1} U_1 + \frac{V_{f2}^{\;2}}{2g}$$

or $$H = \frac{1}{g} V_{w1} U_1 + \frac{C^2 V_{f1}^{\;2}}{2g} \qquad [\because V_{f2} = C V_{f1}]$$

or $$H = \frac{1}{g} \times V_{f1} \cot\alpha_1 \times V_{f1} \cot\alpha_1 + \frac{C^2 V_{f1}^{\;2}}{2g}$$

or $$H = \frac{1}{2g} V_{f1}^{\;2} \left[\frac{2 + C^2 \tan^2\alpha_1}{\tan^2\alpha_1} \right]$$

or $$V_{f1} = \sqrt{2gH} \frac{\tan\alpha_1}{\sqrt{2 + C^2 \tan^2\alpha_1}}$$

or $$\frac{V_{f1}}{\tan\alpha_1} = \sqrt{\frac{2gH}{2 + C^2 \tan^2_{\alpha_1}}}$$

or $$U = \sqrt{\frac{2gH}{2 + C^2 \tan^2_{\alpha_1}}}$$

Hydraulic efficiency is given by

$$\eta_h = \frac{V_{w1} U_1}{gH}$$

$$= \frac{\dfrac{V_{f1}}{\tan\alpha_1} \times \dfrac{V_{f1}}{\tan\alpha_1}}{g \times \dfrac{1}{2g} V_{f1}^{\;2} \left[\dfrac{2 + C^2 \tan^2\alpha_1}{\tan^2\alpha_1} \right]}$$

or, $$\eta_h = \frac{2}{2 + C^2 \tan^2 \alpha_1}$$

Q4.

A Kaplan turbine operating under a net head of 20 m develops 16 MW with an overall efficiency of 80%. The diameter of the runner is 4.2 m, while the hub diameter is 2 m and the dimensionless specific speed is 3 rad. If the hydraulic efficiency is 90%, calculate the inlet and exit angles of the runner blades at the mean blade radius if the flow leaving the runner is purely axial.

Solution

$$\text{Power available} = \frac{\text{Power delivered}}{\text{overall efficiency}}$$

Therefore,

$$10^3 \times \dot{Q} \times 9.81 \times 20 = \frac{16 \times 10^6}{0.8}$$

or $\dot{Q} = 101.94 \text{ m}^3\text{/s}$

Dimensionless specific speed

$$K_{ST} = \frac{NP^{1/2}}{\rho^{1/2}\left(gH\right)^{5/4}}$$

Therefore, $$3 = \frac{N \times \left(16 \times 10^6\right)^{1/2}}{\left(1000\right)^{1/2}\left(9.81 \times 20\right)^{5/4}}$$

which gives $N = 17.41$ rad/s

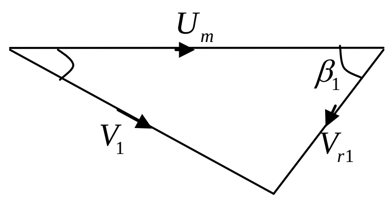
Blade speed at mean radius

$$U_m = 17.41 \times \frac{\left(4.2 + 2\right)}{2 \times 2} = 26.98 \text{ m/s}$$

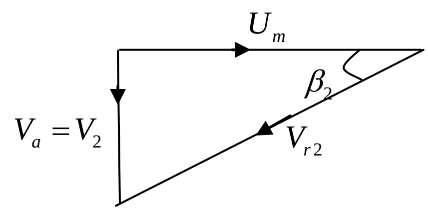
Again $V_{w1}U_m = gH \times \eta_h$

or $$V_{w1} = \frac{gH\eta_h}{U_m}$$

$$= \frac{9.81 \times 20 \times 0.9}{26.98} = 6.54 \text{ m/s}$$



Inlet velocity triangle



Outlet velocity triangle

From inlet velocity diagram

$$\tan\beta_1 = \frac{9.51}{\left(26.98 - 6.54\right)}$$

which gives $\beta_1 = 25°$

From outlet velocity diagram

$$\tan\beta_2 = \frac{9.51}{26.98}$$

which gives $\beta_2 = 19.4°$

Q5.

A conical type draft tube attached to a Francis turbine has an inlet diameter of 3 m and its area at outlet is 20 m^2. The velocity of water at inlet, which is 5 m above tail race level, is 5 m/s. Assuming the loss in draft tube equals to 50 % of velocity head at outlet, find (i) the pressure head at the top of the draft tube, (ii) the total head at the top of the draft tube taking tail race level as datum, and (iii) power lost in draft tube.

Solution

Velocity of water at outlet from draft tube

$$= 5 \times \pi \times \frac{3^2}{4} \times \frac{1}{20} = 1.77 \text{ m/s}$$

(i) Let p_1 be the pressure at inlet to the draft tube. Applying energy equation between the inlet and outlet of the draft tube, we have

$$\frac{p_1}{\rho g} + \frac{5^2}{2\times 9.81} + 5 = 0 + \frac{(1.77)^2}{2\times 9.81} + 0 + 0.5\times\frac{(1.77)^2}{2\times 9.81}$$

or $$\frac{p_1}{\rho g} = 0 - \frac{1}{2\times 9.81}\left[25 - 1.5\times(1.77)^2\right] - 5$$

or $$\frac{p_1}{\rho g} = -6.03 \text{ m}$$

(ii)The total head at inlet

$$= 1.5\times\frac{(1.77)^2}{2\times 9.81} = 0.24 \text{ m}$$

(ii) Head lost in draft tube

$$= 0.5\times\frac{(1.77)^2}{2\times 9.81} = 0.08 \text{ m}$$

Q6.

Calculate the least diameter of impeller of a centrifugal pump to just start delivering water to a height of 30 m, if the inside diameter of impeller is half of the outside diameter and the manometric efficiency is 0.8. The pump runs at 1000 rpm.

Solution

Least diameter of impeller for a pump to just start would correspond to a situation when centrifugal head developed

$$= \frac{\text{static lift}}{\text{manometric efficiency}}$$

Therefore, $$\frac{U_2^2 - U_1^2}{2g} = \frac{30}{0.8} = 37.5 \text{ m}$$

(subscripts 1 and 2 refer to the inlet and outlet of the impeller).

Again, $$\frac{U_2}{U_1} = \frac{D_2}{D_1} = 2$$

Hence, $$\frac{4U_1^2 - U_1^2}{2g} = 37.5$$

or $$U_1 = \sqrt{2\times 9.81\times 37.5/3} = 15.66 \text{ m/s}$$

$$U_2 = 2\times 15.66 = 31.32 \text{ m/s}$$

Hence least diameter of impeller

$$D = \frac{60 \times 31.32}{\pi \times 1000} = 0.6 \text{ m}$$

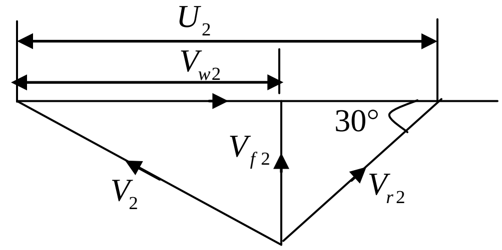
Q7.

A centrifugal pump is required to work against a head of 20 m while rotating at the speed of 700 rpm. If the blades are curved back to an angle of 30° to the tangent at outlet tip and velocity of flow through impeller is 2 m/s, calculate the impeller diameter when (i) all the kinetic energy at impeller outlet is wasted and (ii) when 50% of this energy is converted into pressure energy in pump casing.

Solution

From the outlet velocity triangle

$$V_{w2} = U_2 - V_{f2} \cot \beta_2$$
$$= U_2 - 2\cot 30° = U_2 - 3.46$$



Energy given to the fluid per unit weight

$$= \frac{V_{w2}U_2}{g} = \frac{(U_2 - 3.46)U_2}{g}$$

(a) Under the situation when the entire kinetic energy at impeller outlet is wasted

$$20 = \frac{(U_2 - 3.46)U_2}{g} - \frac{V_2^2}{2g}$$

From the outlet velocity triangle

$$V_2^2 = V_{f2}^2 + V_{w2}^2 = 4 + (U_2 - 3.46)^2$$

Therefore,

$$20 = \frac{(U_2 - 3.46)U_2}{g} - \frac{4 + (U_2 - 3.46)^2}{2g}$$

or $\quad U_2^2 - 18.97 = 2 \times 9.81 \times 20$

which gives $\quad U_2 = 20.21$ m/s

Impeller diameter is

$$D_2 = \frac{60U_2}{\pi N} = \frac{60 \times 20.21}{\pi \times 700} = 0.55 \text{ m}$$

(b) When 50% of the kinetic energy at impeller outlet is converted into pressure energy in pump casing, we can write

$$20 = \frac{(U_2 - 3.46)U_2}{g} - \frac{1}{2}\left\{\frac{4 + (U_2 - 3.46)^2}{2g}\right\}$$

or $\quad 3U_2^2 - 6.92U_2 - 800.77 = 0$

The feasible solution is $U_2 = 17.53$ m/s

Hence, $\quad D_2 = \dfrac{60 \times 17.53}{\pi \times 700} = 0.48$ m