Introduction to Laminar Boundary Layer

Q1. Choose the correct answer

(i) If x is the distance measured from the leading edge of a flat plate, the laminar boundary layer thickness varies as

(a) x

(b) $x^{1/2}$

(c) $x^{-1/2}$

(d) $x^{-4/5}$

[*Ans.(*b)]

(ii) in the entrance region of a pipe, the boundary layer grows and the inviscid core accelerates. This is accompanied by a

(a) pressure pulse

(b) constant pressure gradient

(c) rise in pressure

(d) fall in pressure in the flow direction

[*Ans.(*d)]

(iii) Separation in flow past a solid object is caused by

(a) a favourable (negative) pressure gradient

(b) an adverse (positive) pressure gradient

(c) the boundary layer thickness reducing to zero

(d) a reduction of pressure to vapour pressure

[*Ans.(*b)]

Q2.

Fluid flows with a free stream velocity of U_∞ over a flat plate. The leading edge of the plate is located at x=0. The velocity profiles in the boundary layer having thickness δ are as follows:

(a) $\dfrac{u}{U_\infty} = 2\dfrac{y}{\delta} - \left(\dfrac{y}{\delta}\right)^2$

(b) $\dfrac{u}{U_\infty} = \dfrac{3}{2}\dfrac{y}{\delta} - \dfrac{1}{2}\left(\dfrac{y}{\delta}\right)^3$

(c) $\dfrac{u}{U_\infty} = \dfrac{y}{\delta} - \dfrac{1}{2}\left(\dfrac{y}{\delta}\right)^2$

where y is the transverse distance measured from the plate.

Check whether the boundary conditions satisfied by the profiles or not and select in order of merit the best to worst profile for a laminar boundary layer.

Solution

For flow over a flat plate, the boundary conditions are

(i) at $y = 0$, $u = 0$ (no-slip condition at the plate)

(ii) at $y = \delta$, $u = U_\infty$ (free stream velocity at the edge of the boundary layer)

(iii) at $y=\delta$, $\frac{\partial u}{\partial y}=0$ (zero shear stress at the edge of the boundary layer)

(iv)at $y=0$, $\frac{\partial^2 u}{\partial y^2}=0$ (from momentum equation at $y=0$)

(a) $\frac{u}{U_\infty}=2\frac{y}{\delta}-\left(\frac{y}{\delta}\right)^2$

At $y=0$, $u=0$

At $y=\delta$, $u=U_\infty$

At $y=\delta$, $\frac{\partial u}{\partial y}=0$

At $y=0$, $\frac{\partial^2 u}{\partial y^2}=\frac{2}{\delta^2}$

The velocity profile $\frac{u}{U_\infty}=2\frac{y}{\delta}-\left(\frac{y}{\delta}\right)^2$ satisfies essential boundary conditions (i)-(iii).

(b) $\frac{u}{U_\infty}=\frac{3}{2}\frac{y}{\delta}-\frac{1}{2}\left(\frac{y}{\delta}\right)^3$

At $y=0$, $u=0$

At $y=\delta$, $u=U_\infty$

At $y=\delta$, $\frac{\partial u}{\partial y}=0$

At $y=0$, $\frac{\partial^2 u}{\partial y^2}=0$

The velocity profile $\frac{u}{U_\infty}=\frac{3}{2}\frac{y}{\delta}-\frac{1}{2}\left(\frac{y}{\delta}\right)^3$ satisfies all the above boundary conditions.

Feasible and smoothest

(c) $\frac{u}{U_\infty}=\frac{y}{\delta}-\frac{1}{2}\left(\frac{y}{\delta}\right)^2$

At $y=0$, $u=0$

At $y=\delta$, $u=\frac{U_\infty}{2}$

The velocity profile $\frac{u}{U_\infty}=\frac{y}{\delta}-\frac{1}{2}\left(\frac{y}{\delta}\right)^2$ though satisfies the no slip condition at the plate, but does not satisfy the important boundary condition (ii) at the boundary layer free stream interface. Hence it is not feasible.

Both the profile $\frac{u}{U_\infty} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2$ and $\frac{u}{U_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$ are feasible as they satisfy all three essential boundary conditions. However, velocity profile $\frac{u}{U_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$ is the best profile, as it satisfies all four of the required boundary conditions, whereas $\frac{u}{U_\infty} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2$ does not satisfies the condition (iv).

Q3.
For flow over a body of a given shape, a scaling estimation for the boundary layer thickness is obtained as $\frac{\delta}{x} \sim \mathrm{Re}_x^{-1/5}$. If the free stream velocity changes from 9 m/s to 25 m/s (keeping all other conditions unaltered), then what is the percentage change in wall shear stress (at a given x)? Is it an increment or a decrement?

Solution

It is given that $$\frac{\delta}{x} \sim \mathrm{Re}_x^{-1/5} \sim \left(\frac{U_\infty x}{\nu}\right)^{-1/5}$$

or $$\delta \sim \frac{x^{4/5}\nu^{1/5}}{U_\infty^{1/5}}$$

From the scaling estimation, one can write

$$\tau_w \sim \mu \frac{U_\infty}{\delta}$$

$$\sim \mu \frac{U_\infty U_\infty^{1/5}}{x^{4/5}\nu^{1/5}} \sim \mu \frac{U_\infty^{6/5}}{x^{4/5}\nu^{1/5}}$$

Since other than free stream velocity, all other conditions unaltered, the ratio of shear stress at a given x, can be expressed as

$$\frac{\tau_{w2}}{\tau_{w1}} = \left(\frac{U_{\infty 2}}{U_{\infty 1}}\right)^{6/5}$$

Substituting the values of the free stream velocities, we obtain

$$\frac{\tau_{w2}}{\tau_{w1}} = \left(\frac{25}{9}\right)^{6/5} = 3.4$$

or $$\frac{\tau_{w2} - \tau_{w1}}{\tau_{w1}} = 2.4$$

The change in wall shear stress is 240% and this is an increment.