Conservation Equations in Fluid Flow

Q1. **Choose the correct answer**

(i) Mathematical statement of Reynolds transport theorem is given by

(a) $\left.\frac{dN}{dt}\right|_{\text{system}} = \frac{\partial}{\partial t}\int_{CS} \eta\vec{V}.\, d\vec{A} + \int_{C\forall} \eta\rho d\forall$

(b) $\left.\frac{dN}{dt}\right|_{\text{system}} = \frac{\partial}{\partial t}\int_{CS} \eta\rho\vec{V}.d\vec{A} + \int_{C\forall} \eta\rho d\forall$

(c) $\left.\frac{dN}{dt}\right|_{\text{system}} = \frac{\partial}{\partial t}\int_{CV} \eta\rho d\forall + \int_{CS} \eta\rho\vec{V}.\, d\vec{A}$

(d) $\left.\frac{dN}{dt}\right|_{\text{system}} = \frac{\partial}{\partial t}\int_{CV} \eta\rho d\forall - \int_{CS} \eta\rho\vec{V}.\, d\vec{A}$

where N is an extensive property, η is the property per unit mass, $\vec{V}$ is the velocity vector and $d\vec{A}$ is the elemental area vector on the control surface.

[*Ans.(*c)]

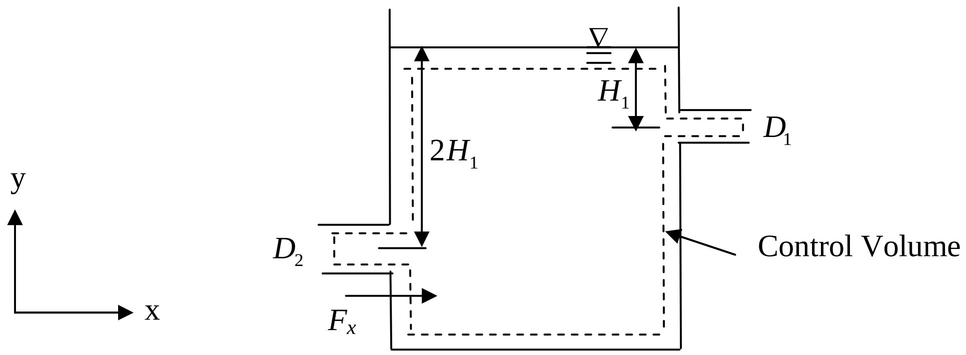
(ii) A fish tank is being carried on a car moving with constant-horizontal acceleration. The level of water will

(a) remain unchanged
(b) rise on the front side of the tank only
(c) rise on the front side of the tank and fall on the back side
(d) rise on the back side of the tank and fall on the front side

[*Ans.(*d)]

Q2.

A tank as shown in the figure below has a nozzle of exit diameter D_1 at a depth H_1 below the free surface. At the side opposite to that of nozzle 1, another nozzle of diameter D_2 is attached to the tank at a depth $2H_1$. Neglecting the frictional effects, find the diameter D_2 in terms of D_1 so that the net horizontal force on the tank is zero.



Solution

A fixed control volume as shown by the dashed line in the above figure is considered for the analysis.

For nozzle 1, applying Bernoulli's equation between a point on the free surface and nozzle exit along a streamline, we have

$$\frac{p_{atm}}{\rho g}+\frac{\cancel{V^2}^{\,0}}{2g}+H_1=\frac{p_{atm}}{\rho g}+\frac{V_1^2}{2g}+0$$

or $$V_1=\sqrt{2gH_1}$$

and mass flow rate $$\dot{m}_1=\rho A_1V_1=\rho\frac{\pi}{4}D_1^2\sqrt{2gH_1}$$

For nozzle 2, applying Bernoulli's equation between a point on the free surface and nozzle exit along a streamline, we have

$$\frac{p_{atm}}{\rho g}+\frac{\cancel{V^2}^{\,0}}{2g}+2H_1=\frac{p_{atm}}{\rho g}+\frac{V_2^2}{2g}+0$$

or $$V_2=\sqrt{4gH_1}=2\sqrt{gH_1}$$

and mass flow rate $$\dot{m}_2=\rho A_2V_2=\rho\frac{\pi}{4}D_2^2 2\sqrt{gH_1}$$

Let F_x be the horizontal force in the positive direction of x on the fluid mass in the control volume inscribing the tank as shown in the figure. Applying the momentum theorem for the control volume, we get

$$\begin{aligned}F_x&=\dot{m}_1V_1+\dot{m}_2\left(-V_2\right)\\&=\left[\rho\frac{\pi}{4}D_1^2\sqrt{2gH_1}\sqrt{2gH_1}\right]-\left[\rho\frac{\pi}{4}D_2^2 2\sqrt{gH_1}2\sqrt{gH_1}\right]\\&=\rho\pi gH_1\left(\frac{D_1^2}{2}-D_2^2\right)\end{aligned}$$

It is given that the net horizontal force acting on the tank is zero, and therefore, we obtain

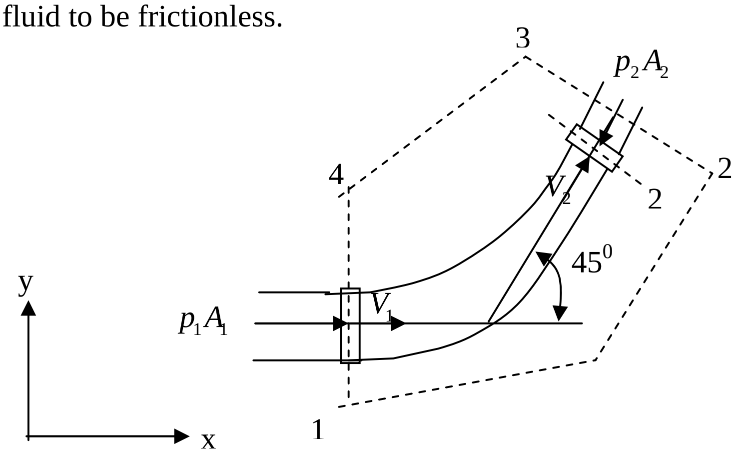
$$\rho\pi gH_1\left(\frac{D_1^2}{2}-D_2^2\right)=0$$

from which we get

$$D_1=\sqrt{2}D_2$$

Q3.

Example 5.3 A 45° reducing pipe-bend in a horizontal plane (shown in the figure below) has an inlet diameter of 600 mm and outlet diameter of 300 mm. The pressure at the inlet is 140 kPa gauge and rate of flow of water through the bend is 0.425 m^3/s. Neglecting friction, calculate the net resultant horizontal force exerted by the water on the bend. Assume uniform conditions with straight and parallel streamlines at inlet and outlet and the fluid to be frictionless.



Solution

A fixed control volume 12341 as shown by the dashed line in the above figure is considered for the analysis.

The inlet velocity is

$$V_1 = \frac{Q}{A_1} = \frac{0.425}{\frac{\pi}{4}(0.6)^2} = 1.503 \text{ m/s}$$

The outlet velocity is

$$V_2 = \frac{Q}{A_2} = \frac{0.425}{\frac{\pi}{4}(0.3)^2} = 6.01 \text{ m/s}$$

Applying Bernoulli's equation along a streamline connecting sections 1 and 2, we have

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g}$$

or
$$\frac{140\times10^3}{1000\times9.81} + \frac{1.503^2}{2\times9.81} = \frac{p_2}{1000\times9.81} + \frac{6.01^2}{2\times9.81}$$

or,
$$p_2 = 123.1\times10^3 \text{ Pa} = 123.1 \text{ kPa}$$

Applying the momentum theorem to the control volume 12341, we get

$$p_1A_1 - p_2A_2\cos 45° + F_x = \rho Q\left(V_2\cos 45° - V_1\right)$$

and
$$-p_2A_2\sin 45° + F_y = \rho Q\left(V_2\sin 45° - 0\right)$$

where F_x and F_y are the forces in the x and y directions exerted by the bend on water in the control volume.

Substituting the respective values in the above two equations, we get

$$140\times10^3\left(\frac{\pi}{4}\right)(0.6)^2-123.1\times10^3\left(\frac{\pi}{4}\right)(0.3)^2\cos45°+F_x=1000\times0.425(6.01\cos45°-1.503)$$

$$-123.1\times10^3\left(\frac{\pi}{4}\right)(0.3)^2\sin45°+F_y=1000\times0.425(6.01\sin45°-0)$$

which give,

$$F_x=-32.26\text{ kN}$$

$$F_y=7.96\text{ kN}$$

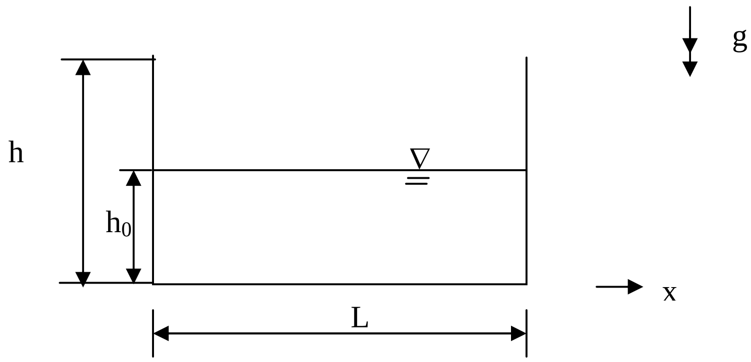
Therefore, the resultant force on the water is

$$F=\sqrt{(32.26)^2+(7.96)^2}=33.23\text{ kN}$$

The resultant force makes an angle of $\tan^{-1}(7.96/32.26)=13.86°$ with the negative direction of *x*-axis. According to Newton's third law, the force exerted by the water on the bend is equal and opposite to the force F.

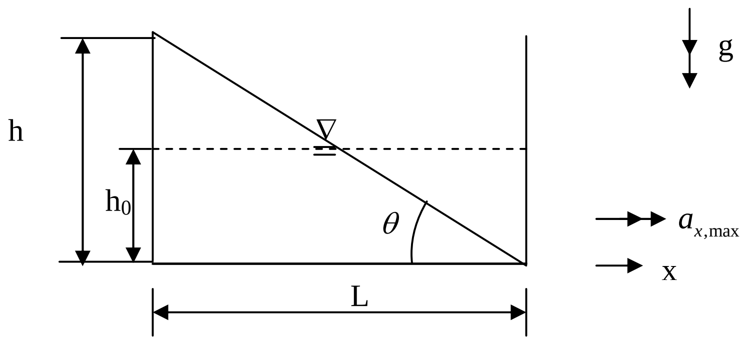
Q4.

An open rectangular tank of length $L=10\text{ m}$, height $h=4\text{ m}$ and width (perpendicular to the plane of the figure below) 1 m is initially half-filled with water. The tank suddenly accelerates along the horizontal direction with an acceleration = 0.5g (where, g is the acceleration due to gravity). Will any water spill out of the tank?



Solution

Let us consider the case that the tank accelerates with a maximum acceleration a_x along the horizontal direction without spilling the water. Since the volume of the water in the tank remains unchanged, the free surface takes the shape as shown in the figure below.



From the principle of relative equilibrium, one can write

$$\tan\theta = \frac{h}{L} = \frac{a_{x,\max}}{g}$$

or
$$\frac{a_{x,\max}}{g} = \frac{4}{10} = 0.4$$

or
$$a_{x,\max} = 0.4g$$

This is the maximum acceleration that can be given without spilling the water. Since the given acceleration is higher than this value, the water will spill out of the tank.