Mechanics

Module VII: Analysis of Beams

Lesson 34: Shear Force and Bending Moment - I

A beam is a slender structural element that can transmit/resist bending. We will analyze beams only under planar bending in the plane of the paper. It will be assumed that

- The beam is loaded by forces transverse to its axis in the plane, and/or by torques perpendicular to the plane
- The transverse deflection at any point is much smaller compared to the length of the beam.

A beam can have different types of support conditions, as shown in Fig. 1. A beam with only pinned and roller support is known as a simply supported beam, while a beam with a built-in end and a free end is known as a cantilever beam. These are depicted in Fig. 2. Given the beam support configuration and the loading, we can determine the support reaction forces/moment (for a determinate beam structure). The next task is to determine the internal forces and moments in the beam.

Consider a beam with transverse loading, as shown in Fig. 3. If we take a section of the beam at x, as shown in the figure, we have the shear force $V(x)$,

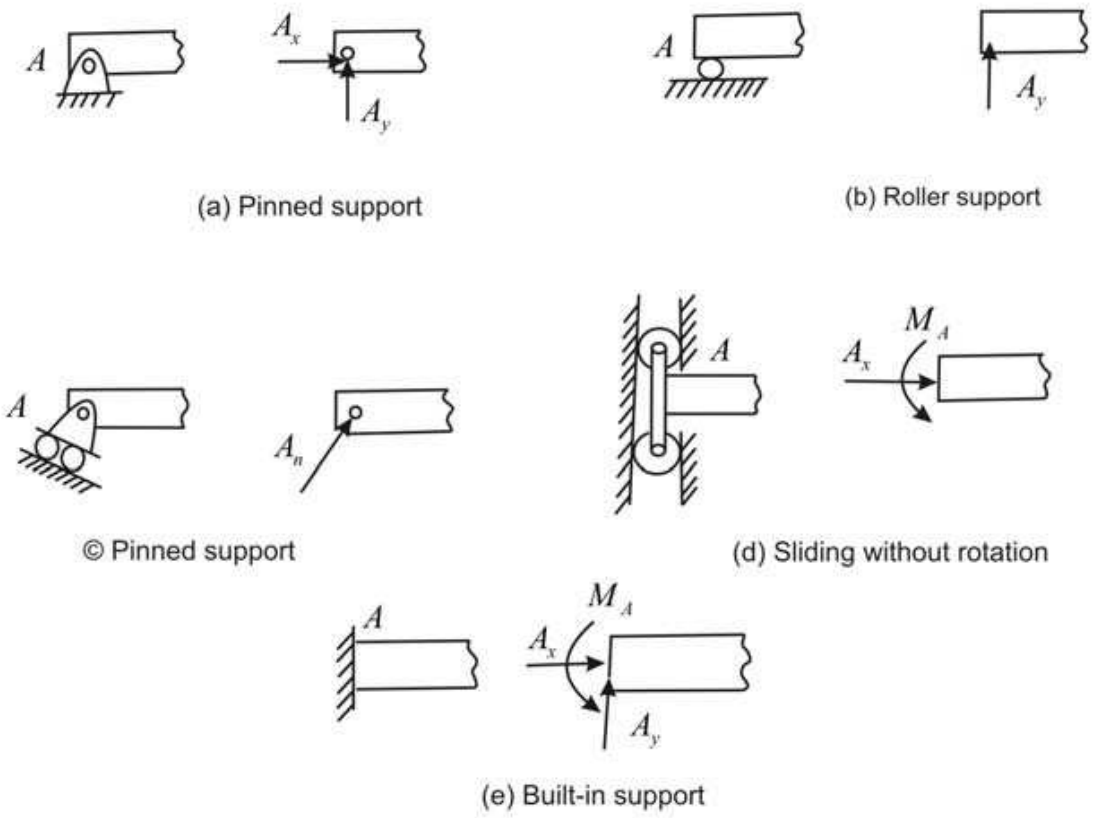


Figure 1:

and bending moment $M(x)$. The sign convention is also shown in the figure. The figures representing the variations of $V(x)$ and $M(x)$ over the length of the beam are known as, respectively, the shear force diagram (SFD) and the bending moment diagram (BMD). The SFD and BMD form the starting point for analysis of stresses and deflection of beams. In order to obtain the SFD and BMD of determinate beam problems, we perform the following two steps

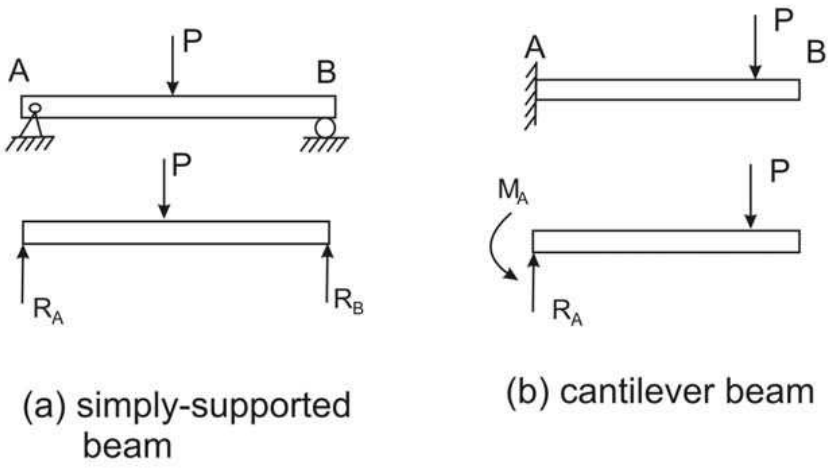


Figure 2:

1. Determination of the support reactions: In this step, for convenience, we replace all distributed forces/moments by their equivalent concentrated force at the centroid of the distribution and determine the support reactions

2. Determination of shear force and bending moment: Reverting back to the force/moment distributions, we can use the method of sections for this calculation.

Problem 1

Draw the shear force and bending moment diagrams for a cantilever beam loaded by a concentrated force P at the free end, as shown in Fig. 4.

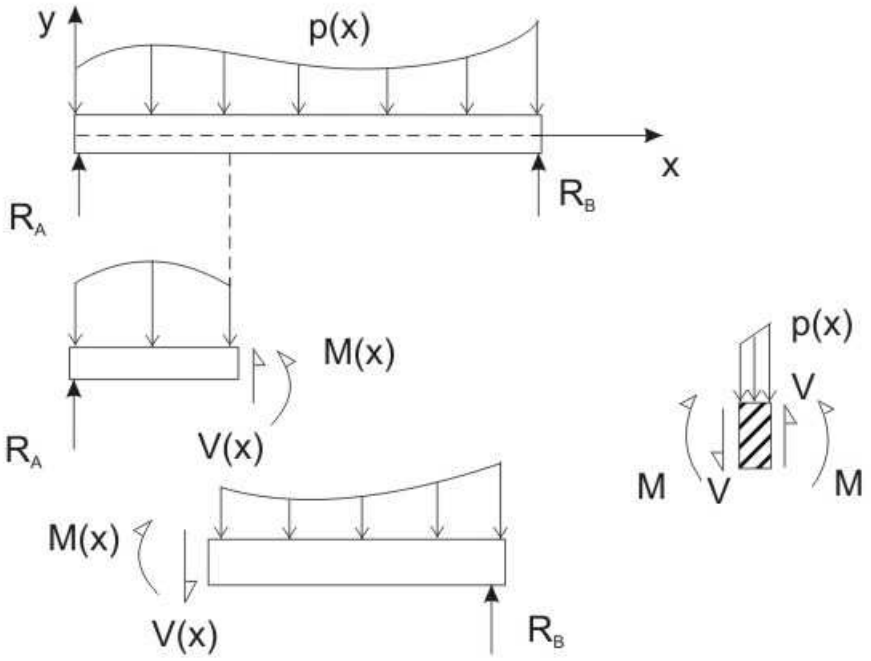


Figure 3:

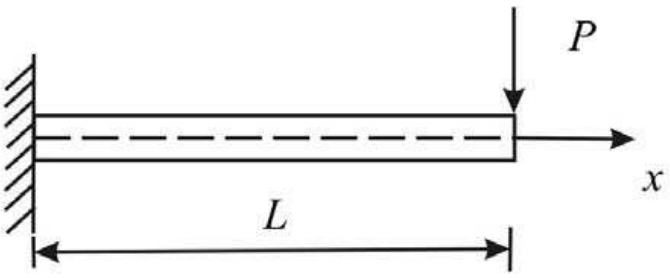


Figure 4:

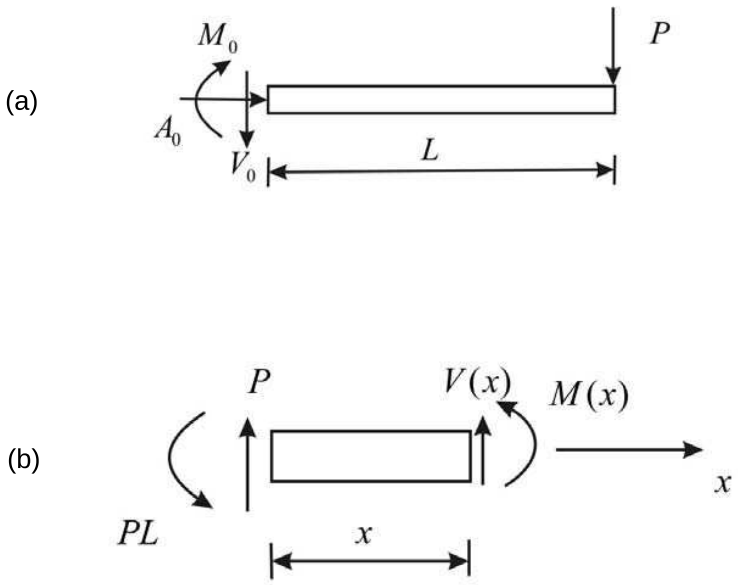


Figure 5:

Solution

Consider the free body diagram (FBD) of the beam as shown in Fig. 5(a). Using the equations of equilibrium

$$\sum F_x = 0 \;\Rightarrow\; A_0 = 0$$

$$\sum F_y = 0 \;\Rightarrow\; -V_0 - P = 0 \;\Rightarrow\; V_0 = -P$$

$$\sum M_0 = 0 \;\Rightarrow\; -M_0 - PL = 0 \;\Rightarrow\; M_0 = -PL$$

Consider the FBD of a section of the beam between $x = 0$ and x as shown in Fig. 5(b). Using the equations of equilibrium

$$\sum F_y = 0 \;\Rightarrow\; V(x) + P = 0 \;\Rightarrow\; V(x) = -P$$

$$\sum M_0 = 0 \;\Rightarrow\; xV(x) + PL + M(x) - PL = 0 \;\Rightarrow\; M(x) = -P(L - x)$$

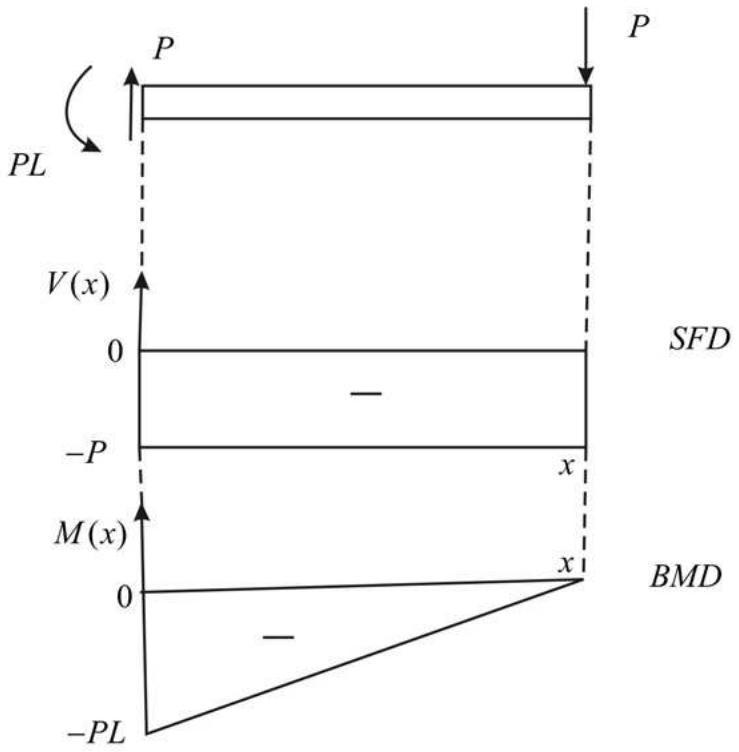


Figure 6:

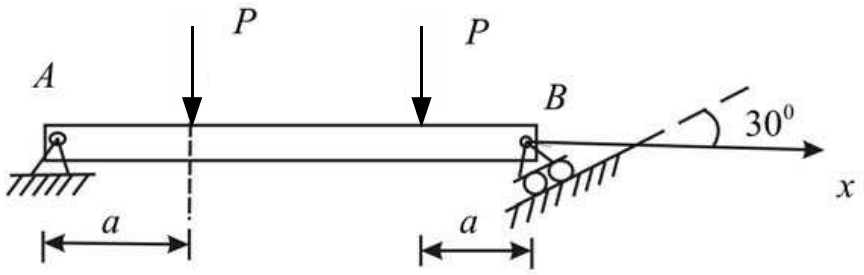


Figure 7:

The SFD and BMD are as shown in Fig. 6. It may be noted that, in the region where there is no external load on the beam, the shear force remains unchanged.

Problem 2

Determine the distribution of shear force and bending moment in a simply supported beam loaded as shown in Fig. 7.

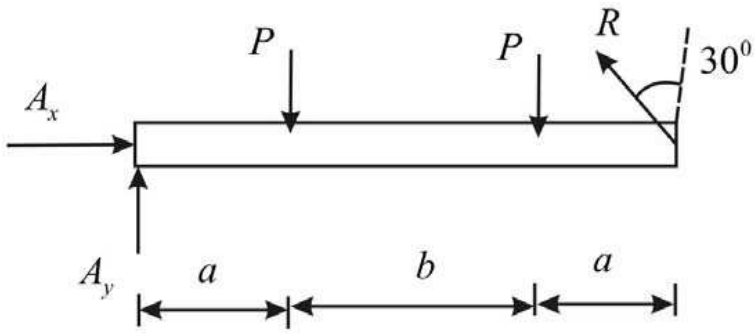


Figure 8:

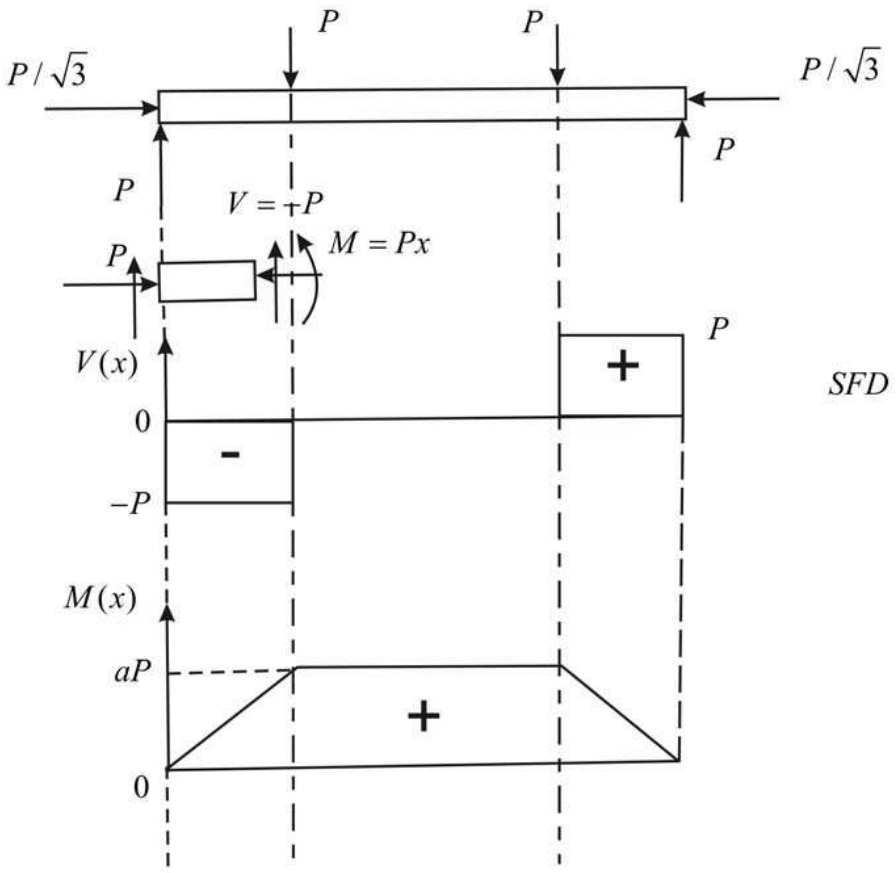


Figure 9:

Solution

The FBD of the beam is as shown in Fig. 8. From the equations of equilibrium we have $A_x = P/\sqrt{3}$, $A_y = P$, and $R = 2P/\sqrt{3}$. Thus, the beam has a constant axial force. The SFD and BMD presented in Fig. 9 can be obtained using the method of sections. It may be noted in this example that, at the location of a concentrated force, the shear force exhibits a discontinuity. Further, over the region in which the shear force is zero, the bending moment

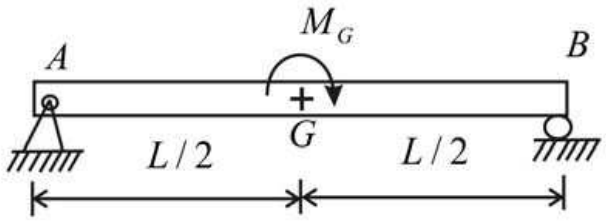


Figure 10:

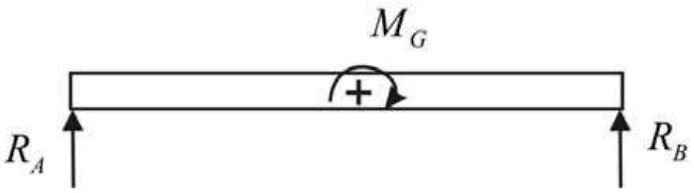


Figure 11:

remains unchanged.

Problem 3

Draw the SFD and BMD for the loaded simplysupported beam shown in Fig. 10.

Solution

The FBD of the beam is shown in Fig. 11. From equilibrium equations, $R_A = -M_G/L = -R_B$. The SFD and BMD are presented in Fig. 12. In the above example, we observe that a concentrated moment leads to a discontinuity in the bending moment at the location of the concentrated moment.

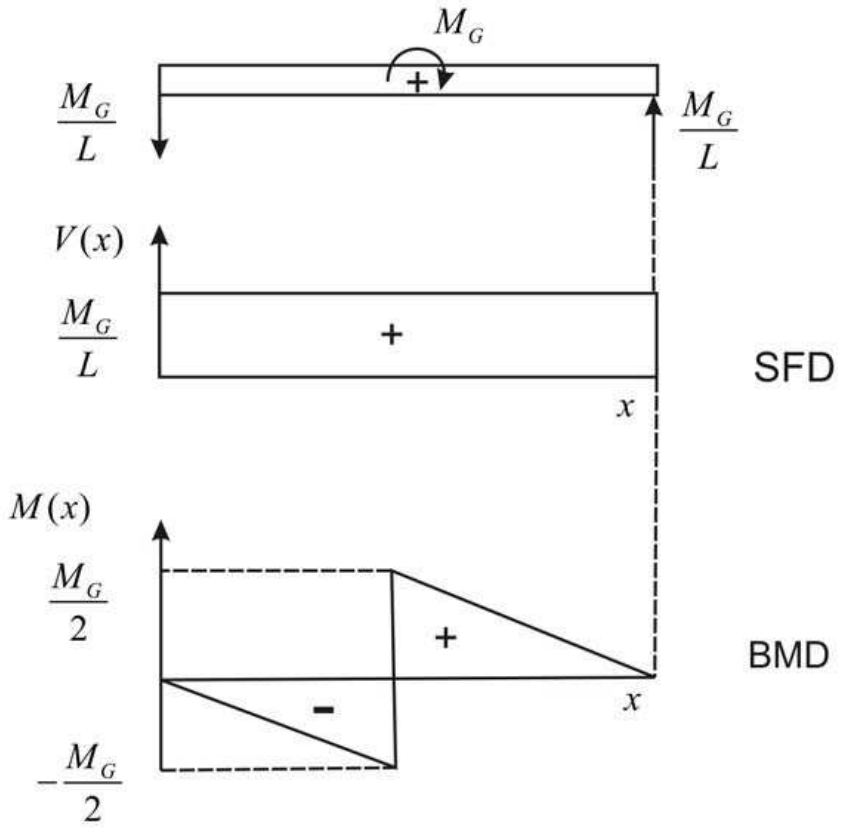


Figure 12: