Mechanics

Module III: Friction

Lesson 16: Friction in Wedges

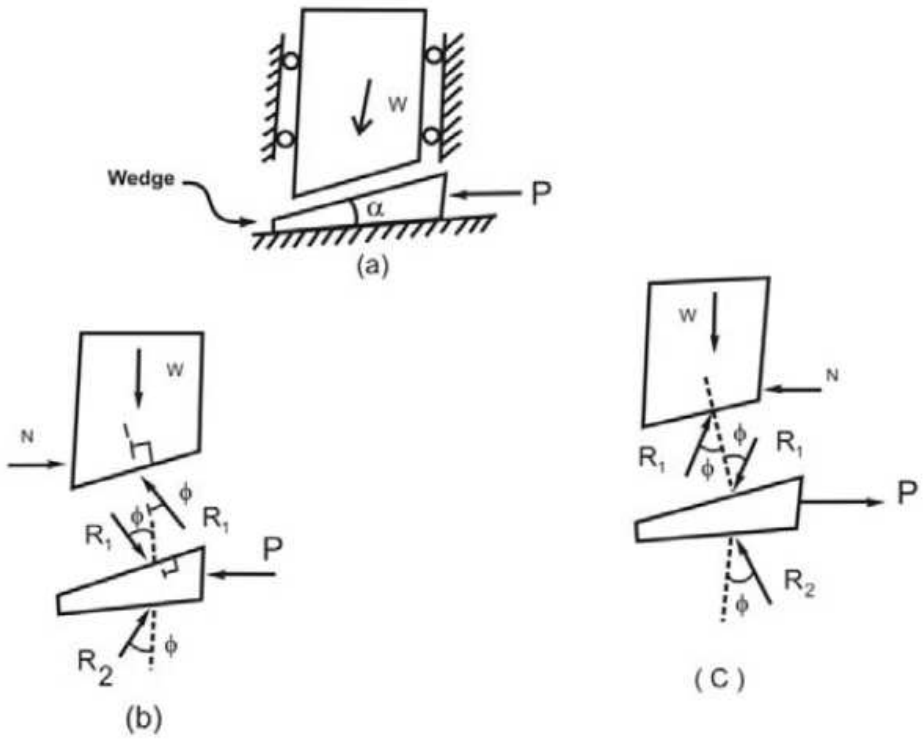


Figure 1:

A wedge is a simple machine that is used for producing large forces or giving small displacements. The key requirement in wedges is the self-locking feature which depends on friction. A wedge, shown in Fig. 1(a), is self-locking when it can remain in equilibrium without any external force P.

Consider a massless wedge being moved (or in impending motion) under a

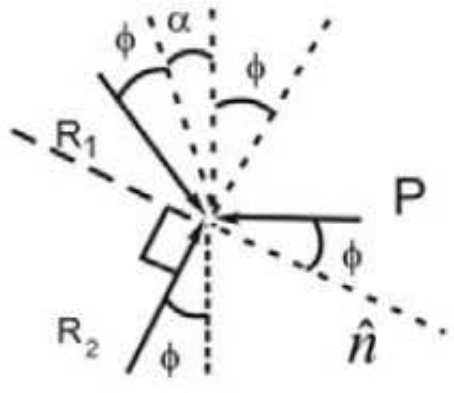


Figure 2:

load, as shown in Fig. 1. In such a situation, all contacting surfaces must be slipping/in impending slippage. The FBDs are shown in Fig. 1(b) for pushing in and Fig. 1(c) for pulling out. In these figures, The force P required to push the wedge in (see Fig. 1(b)) can be determined as follows. From the FBD of the block

$$\sum F_y = 0 \;\Rightarrow\; R_1 \cos(\alpha + \phi) - W = 0 \;\Rightarrow\; R_1 = \frac{W}{\cos(\alpha + \phi)}$$

From the force system on the wedge shown in Fig. 2

$$\sum F_n = 0 \;\Rightarrow\; R_1 \sin(\alpha + 2\phi) - P\cos\phi = 0 \;\Rightarrow\; P = \frac{W \sin(\alpha + 2\phi)}{\cos(\alpha + \phi)\cos\phi}.$$

In order to calculate the force P required to pull out the wedge, consider the FBDs in Fig 1(c). From the FBD of the block

$$\sum F_y = 0 \;\Rightarrow\; R_1 \cos(\phi - \alpha) - W = 0 \;\Rightarrow\; R_1 = \frac{W}{\cos(\phi - \alpha)}$$

From the force system on the wedge shown in Fig. 3

$$\sum F_n = 0 \;\Rightarrow\; R_1 \sin(2\phi - \alpha) + P\cos\phi = 0 \;\Rightarrow\; P = \frac{W \sin(2\phi - \alpha)}{\cos(\phi - \alpha)\cos\phi}.$$

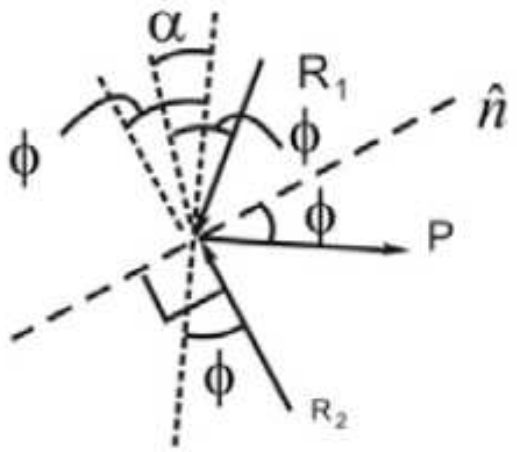


Figure 3:

Now, for self-locking, this pullout force should be positive, i.e., $P > 0$. Hence, $2\phi > \alpha$ for self-locking. Given the coefficient of static friction μ, this condition puts a limit on the wedge angle $\alpha < \tan^{-1}\mu$. When the external force $P = 0$, the reaction forces R_1 and R_2 are collinear. Some extreme possibilities of force configurations are shown in Fig. 4.

Problem 1

A 100 kg door is lifted slightly using a 5° wedge, as shown in Fig. 5. Given the coefficient of friction on all surfaces to be 0.6, determine the force P required to lift the door.

Solution

The FBD of the door (assumed to be almost horizontal) and the wedge in impending sliding is shown in Fig. 6. Here, $\phi = \tan^{-1} 0.6 = 31°$. For the door

$$\sum M_A = 0 \;\Rightarrow\; 2.2F_B \cos(\phi + 5°) - 120g = 0 \;\Rightarrow\; F_B = 660.7 \text{ N}.$$

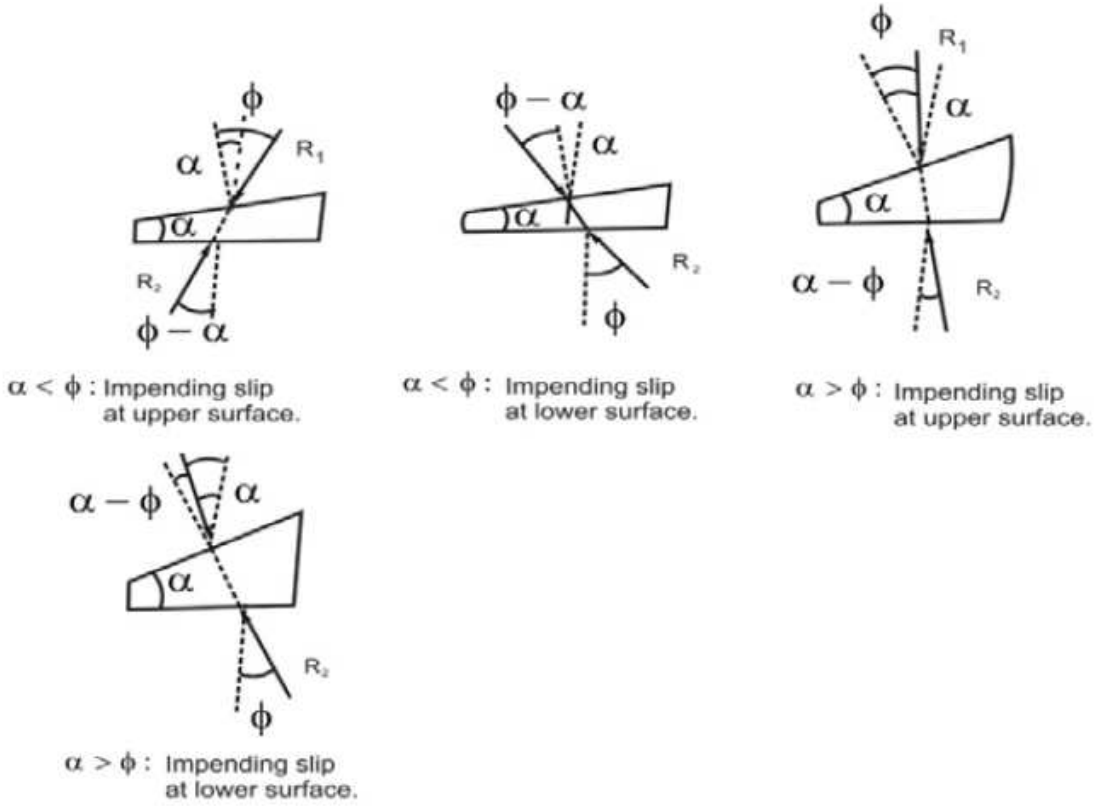


Figure 4:

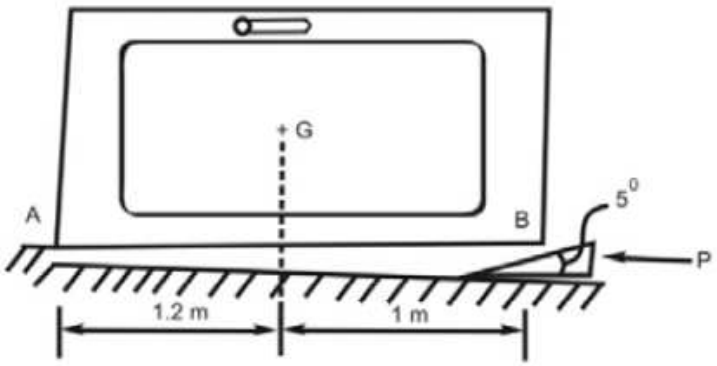


Figure 5:

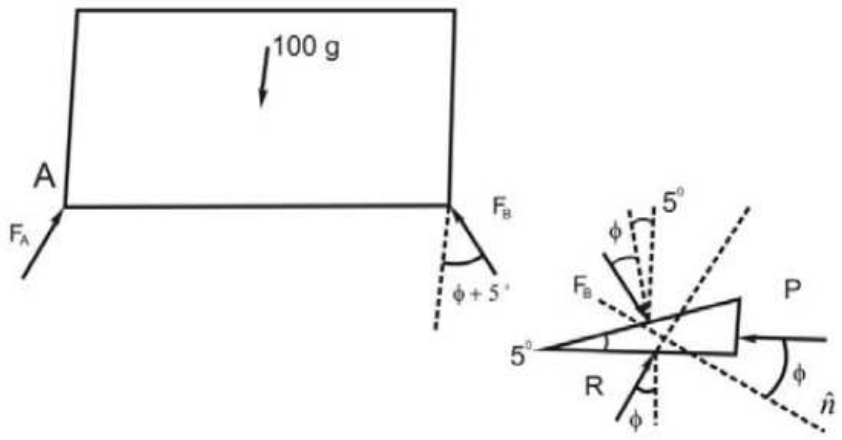


Figure 6:

For the wedge

$$\sum F_n = 0 \;\Rightarrow\; F_B \sin(2\phi + 5^\circ) - P\cos\phi = 0 \;\Rightarrow\; P = 709.56 \text{ N}.$$

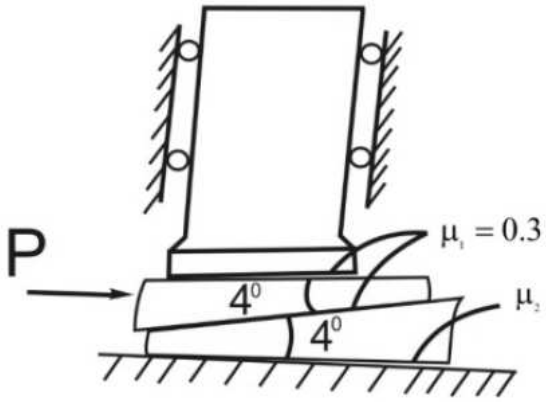


Figure 7:

Problem 2

A block has to be lifted using two 4° wedges, as shown in Fig. 7. Determine the minimum coefficient of friction at the bottom surface of the lower wedge so that the block can be lifted.

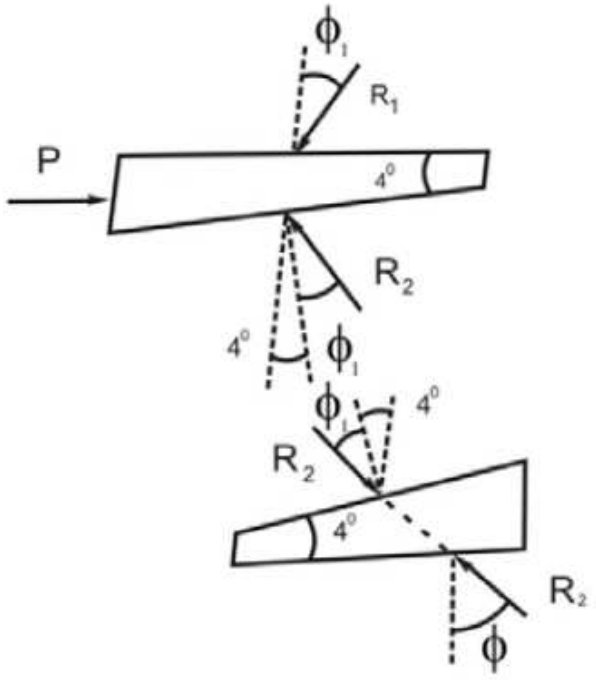


Figure 8:

Solution

The FBD of the wedges at impending slip of the upper block is shown in Fig. 8. Here, $\phi_1 = \tan^{-1} 0.3 = 16.7^\circ$. The lower wedge must not slip at the lower surface for the block to be lifted. Hence, $\theta = \phi_1 + 4^\circ < \tan^{-1} \mu_1 \Rightarrow \mu_1 > \tan 20.7^\circ = 0.378$.