# Vibrations of Structures 

Module V: Vibrations of Plates

## Exercises

1. A square plate of side $a$ is simply-supported at the four edges, and carries a particle of mass $m$ at the center. Determine the eigenfrequencies and eigenfunctions of the plate.
2. A circular plate of radius $a$ is simply supported at the boundary. Determine the dynamic reaction forces at the boundary for different modes of vibration of the plate.
3. An elliptic plate of semi-major axis $a$ and semi-minor axis $b$ is simply supported at the boundary. Determine the approximate eigenfrequencies and modes of vibrations. Plot the variation of the first six eigenfrequencies with the ratio $a / b$ in the range $(1,2)$.


Figure 1: Exercise 4
4. A circular plate of radius $a$ is simply-supported on a circle of radius $b$, as shown in Fig. 1. Determine the optimum ratio $b / a$ for which the plate is most firmly supported in the mode $(0,1)$ (i.e., the corresponding frequency is maximized).
5. A circular plate of radius $a$ is clamped at the boundary $r=a$. A particle of mass $m$ is dropped from a height $h$ exactly on the center of the plate. The particle sticks to the plate. Determine the motion of the plate and the force between the particle and the plate.


Figure 2: Exercise 6
6. An annular plate of inner radius $a$ and outer radius $b$ is clamped at the boundary $r=b$, and clamped to a massless collar (at $r=a$ ) sliding without friction on a guide, as shown in Fig. 2. Determine the eigenfrequencies and eigenfunctions of the system. If the collar is excited by a harmonic force $Q(t)=A \cos \Omega t$, determine the response of the plate.
7. A circular plate of radius $a$ is clamped at the boundary. A constant point force is traveling on a circular path around the center of the plate at a radius $r_{0}$, i.e., $q(r, \phi, t)=Q_{0} \delta\left(r-r_{0}\right) \delta(\phi-\Omega t)$, where $Q_{0}$ is the constant magnitude, and $\Omega$ is the angular speed. Determine the response of the plate. At what values of $\Omega$ will the plate resonate?
8. A square plate of side $a$ is simply supported at the edges on a rigid frame. The frame is given harmonic angular oscillations of circular frequency $\Omega$ about a center line parallel to an edge. Determine the response of the plate.

