MODULE-5: HYDROSTATIC UNITS (PUMPS & MOTORS)

LECTURE- 19: ANALYSIS OF AN AXIAL-PISTON SWASH-PLATE TYPE HYDROSTATIC PUMP (DISCHARGE FLOW CHARACTERISTICS)

Introduction

Description of the Axial-Piston Swash-Plate type hydrostatic pump:

'Axial Inline Linear Piston' or simply called as 'Linear Piston' swash-plate type hydrostatic pumps (Fig.-1) are widely used as the input power source for hydraulic circuitry. As the name suggests, these machines are comprised of a discrete number of pistons, inside a barrel, reciprocate in a sinusoidal fashion for the purposes of displacing the fluid. Due to the discrete nature of design, the flow output of an axial-piston pump is not perfectly smooth and tends to maintain some of the sinusoidal characteristics of the fluid displacement elements themselves. These sinusoidal characteristics are usually referred to as the flow ripple of the pump and are often suspected for generating undesirable vibration and noise [1]

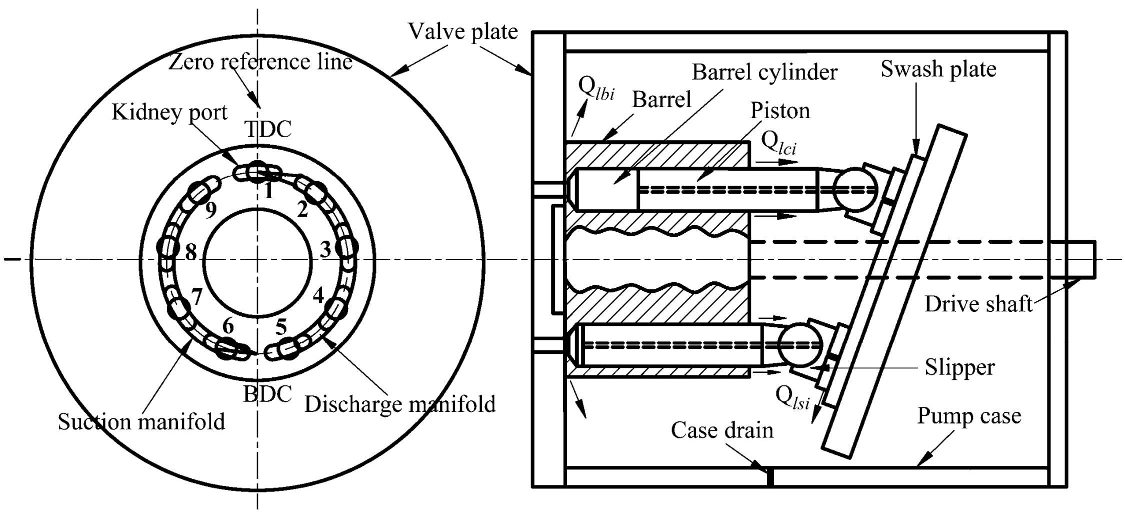

Figure 1: Basic feature of an Inline Linear Axial Piston Swash Plate Type Hydrostatic Unit.

Referring to Fig. 1 the pump consists of several pistons equispaced longitudinally equidistant (on a pcd) and parallel to the central driving shaft in a common cylindrical block, called as barrel. The said shaft rotates the barrel. The rotation of the main barrel forces the pistons to rotate with it. Ends of pistons are connected to shoes with spherical joint. Shoes, called as slipper pads, slide on an inclined swash plate. Springs in the cylindrical barrel force the pistons towards the swash plate. The inclination of the stationary swash plate with respect to the axis of the pump barrel causes the pistons, which are connected with the plate by a spherical ball joint, to reciprocate inside the respective cylindrical bores due to rotation of the barrel along with the pistons. The squeezing action causes delivery to an upstream system at a higher pressure through the discharge manifold [2, 3]. During one revolution, the piston moves inside the barrel from top right to bottom left and back to the starting point. The horizontal distance between the top right and the bottom left are fixed for fixed inclination of swash plate. And the geometric displacement per revolution, called as 'swept volume', remains constant. Thus the name of the pump fixed displacement pump. The end of the piston nearer to the port plate moves closer to the plate during one half of the rotation and away from it during the other half. Two openings on the valve plate covering almost 180 degree angles each are provided for suction and discharge manifolds for the pump. During the movement of the piston over the opening of the discharge manifold, the high pressure fluid is delivered into the discharge manifold and during the piston movement over the suction manifold, fluid inflows from the suction manifold into the piston chamber. By varying the angle of inclination of the swash plate the 'swept volume' can be varied and the pump is made variable displacement.

Two pump flow configurations are considered for the present analysis. For the idealized case, an ideal pump is considered where the leakage is considered to be zero and the fluid is considered to be incompressible. In the real or actual case, leakages in the pump and also the fluid compressibility are considered. For both the idealized case and the actual case, a comparison is made between a nine-piston pump, an eight-piston pump and a seven-piston pump, and conclusions are drawn for understanding the performance of such pumps

Analysis of the Idealized Pump Flow:

Let us consider N number of pistons is in the pump. In this (ideal) case the flow rate generated by each piston is equal to the change of the volume of the piston chamber. If the flow rate generated by each piston is equal to Q_n and instantaneous volume of each piston is V_n then

$$Q_n = \frac{-dV_n}{dt} \quad (1)$$

The maximum linear movement in one direction of the piston is called stroke of the piston and is the same for all the pistons [3]. Let the stroke of each piston be represented by S_p . If the radius of the pitch circle, on which the piston are laid, is considered to be r and the swash plate angle with respect to the vertical axis is α (Fig. 2) , then

$$S_p = 2 \times r \times \tan(\alpha) \quad (2)$$

The maximum volume the fluid can occupy in the piston chamber is the sum of clearance volume and the product of the stroke of the piston and area of cross section of the piston.

$$V_{n.max} = V_c + S_p A_p \quad (3)$$

If the displacement of the piston inside the chamber is y_n , then from simple similar triangle considerations, we get the following for y_n :

$$y_n = r\tan(\alpha)(1 - \cos(\theta_n)) \quad (4)$$

Where θ_n is the angle covered by the nth. piston from the initial position.

The volume of the piston chamber at any instant is given by,

$$V_n = V_c + A_p (S_p - y_n) \quad (5)$$

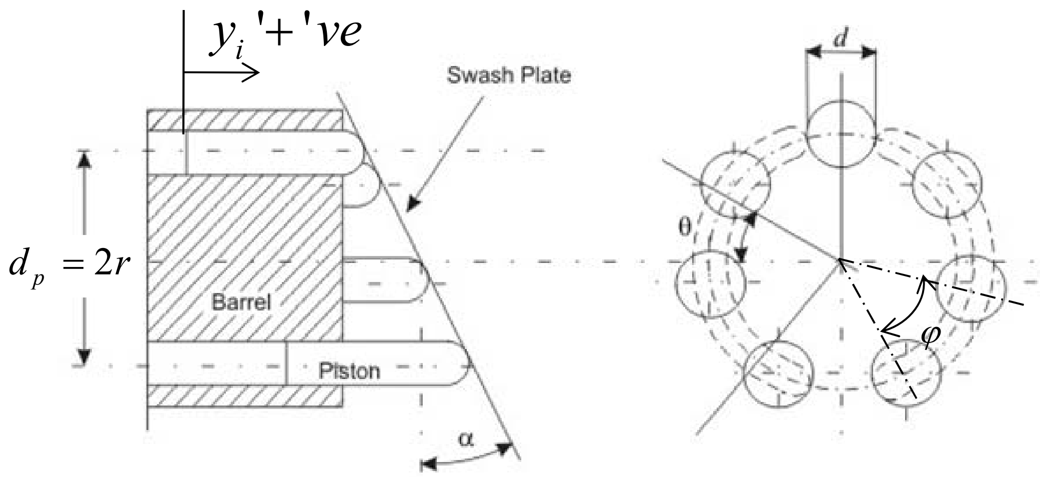

Fig. 2: Schematic view showing geometry.

Therefore,

$$\frac{dV_n}{dt} = -A_p \frac{dy_n}{dt} \tag{6}$$

$$Q_n = \frac{-dV_n}{dt} = A_p \frac{dy_n}{dt} = A_p\, r\, \omega \sin(\theta_n) \tan(\alpha) \tag{7}$$

θ_n is given by,

$$\theta_n = \theta_1 + (n-1)\varphi \tag{8}$$

where , φ = angular separation between the pistons is given by,

$$\varphi = 360^o / N \tag{9}$$

The above flow rate is the flow rate generated by individual pistons. The total flow rate into the suction or discharge manifolds are given by

$$Q = A_p r \tan(\alpha) \omega \sum_{n=1}^{n'} Q_n \tag{10}$$

where the summation is carried out from 1 to n' where n' is the total number of pistons that are instantaneously positioned over the discharge port of the pump. For a pump with even number of pistons this is given by

$$n' = \frac{N}{2} \tag{11}$$

For a pump with odd number of pistons, this quantity is given by,

$$n' = \frac{N \pm 1}{2} \tag{12}$$

± 1 indicates that the number of pistons positioned over the discharge or suction ports is repeatedly fluctuating and depends upon the rotational position of the pump itself.

Using the integral average of the above equation, the nominal discharge flow of the pump may be expressed as,

$$\overline{Q} = \frac{N\, A_p\, r \tan(\alpha) \omega}{2\pi} \int_0^{\pi} \sin(\theta)\, d\theta = \frac{N\, A_p\, r \tan(\alpha) \omega}{\pi} \tag{13}$$

Dividing the general flow equation with the average flow equation derived above we get the normalized flow rate, which is given by

$$\hat{Q}=\frac{\pi}{N}\sum_{n=1}^{n'}\sin(\theta_n) \qquad (14)$$

The above expression's result will be different for even and odd number of pistons. This is the cause for the notion that frequencies and amplitudes of flow ripples generated by even number of pistons are different from those of odd number of pistons (See Lecture note 17).

It can be shown that,

$$\sum_{n=1}^{n'}\sin(\theta_n)=\sin\left(\frac{\pi n'}{N}\right)cosec\left(\frac{\pi}{N}\right)\sin\left(\theta_1+\pi\frac{n'-1}{N}\right) \qquad (15)$$

From simple trigonometric considerations, this reduces for even and odd cases as follows. For even number of pistons,

$$Q_e=\frac{\pi}{N}cosec\left(\frac{\pi}{N}\right)\cos\left(\theta_1-\frac{\pi}{N}\right) \qquad (16)$$

Where, $0\leq\theta_1\leq2\frac{\pi}{N}$

For odd number of pistons,

$$Q_o=\frac{\pi}{N}cosec\left(\frac{\pi}{2N}\right)\cos\left(\theta_1-\frac{\pi}{2N}\right) \qquad (17)$$

Where, $0\leq\theta_1\leq\frac{\pi}{N}$

For even number of pistons, the normalized amplitudes are given by,

$$\Delta\widehat{Q_e}=\frac{\pi}{N_e}\tan\left(\frac{\pi}{2N_e}\right) \qquad (18)$$

and the period of pulse is

$$T_e=\frac{2\pi}{N_e} \qquad (19)$$

For odd number of pistons,

$$\Delta \widehat{Q}_o = \frac{\pi}{2 N_o} \tan\left(\frac{\pi}{4 N_o}\right) \tag{20}$$

and pulse width is

$$T_o = \frac{\pi}{N_o} \tag{21}$$

Analysis of Actual Pump Flow:

In the analysis of the actual pump flow, various leakages and also the compressibility of the fluid is considered [1].

The flow rate from the orifice equation is given by,

$$Q_n = sign(P_n - P_d) C_d A_{o_n} \sqrt{\frac{2}{\rho} |P_n - P_d|} \tag{22}$$

Where, p_n is the instantaneous fluid pressure within the nth. piston chamber, p_d is the discharge pressure of the pump, C_d is the discharge coefficient, A_{o_n} is the instantaneous cross-sectional area of the discharge flow, and ρ is the fluid density. A_{o_n}

The cross sectional area is to be calculated knowing the instantaneous port geometry as shown in figures -3 , 4 and 5.

The total discharge flow of the pump is equal to the net flow generated from each piston chamber instantaneously positioned over the discharge port. Dividing this sum total by the nominal flow provided in equation 5.19-13 above, we get the normalized flow as,

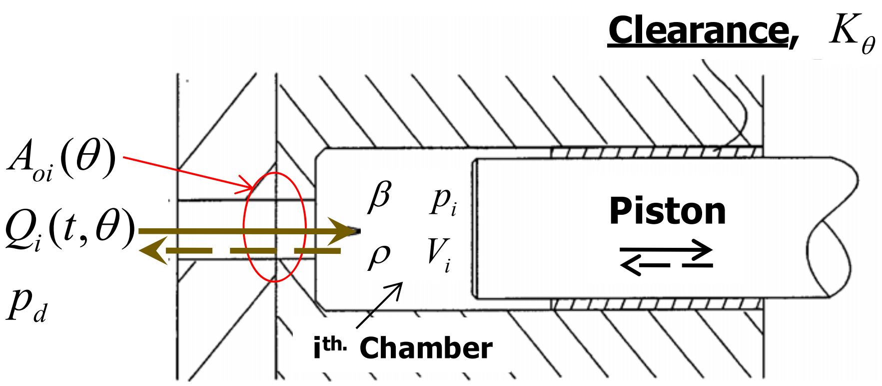

Figure 3 : A typical Piston inside Barrel cylinder.

$$Q = \frac{\pi}{NA_p \mathrm{r} \tan(\alpha)\omega} \sum_{n=1}^{n'} sign(P_n - P_d) C_d A_{o_n} \sqrt{\frac{2}{\rho}|P_n - P_d|} \tag{23}$$

The value of n' is given previously in the above equations for even and odd cases.

So for the calculation of the flow rates, the pressures in each of the pistons are required at each of the angles of rotation of the piston.

From the equation of the bulk modulus:

$$\frac{dP_n}{dt} = \frac{-\beta}{V_n}\left(Q_n + Q_{leak_n} + \frac{dV_n}{dt}\right) \tag{24}$$

Q_{leak} is the leakage that occurs due to the clearances (see Fig.-2) between the piston and bore and/or any other leak paths that exist in the design of the piston chamber. If the leakage flow is modeled as a laminar flow, then leakage flow rate is given by,

$$Q_{leak_n} = K P_n \tag{25}$$

K is the leakage coefficient which may vary depending on the capillary leakage passage length. It is to be found experimentally.

So instantaneous rate of pressure in the nth. piston is given by,

$$\frac{dP_n}{d\theta_n} = -\beta \frac{sign(P_n - P_d) C_d A_{o_n} \sqrt{\frac{2}{\rho}|P_n - P_d|} + K P_n - A_p r \tan(\alpha)\sin(\theta_n)\omega}{(V_o + A_p r \tan(\alpha)\cos(\theta_n))\omega} \tag{26}$$

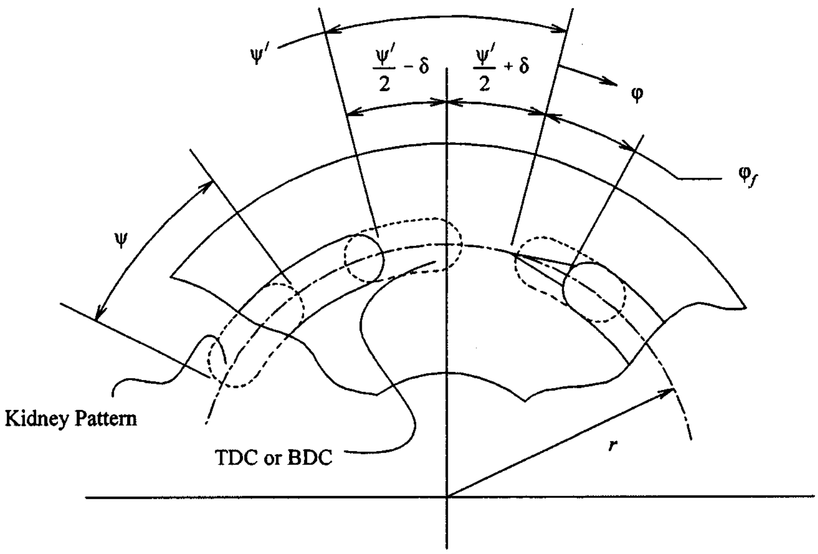

Figure 4 : Typical valve Plate (Kidney Pattern Port).

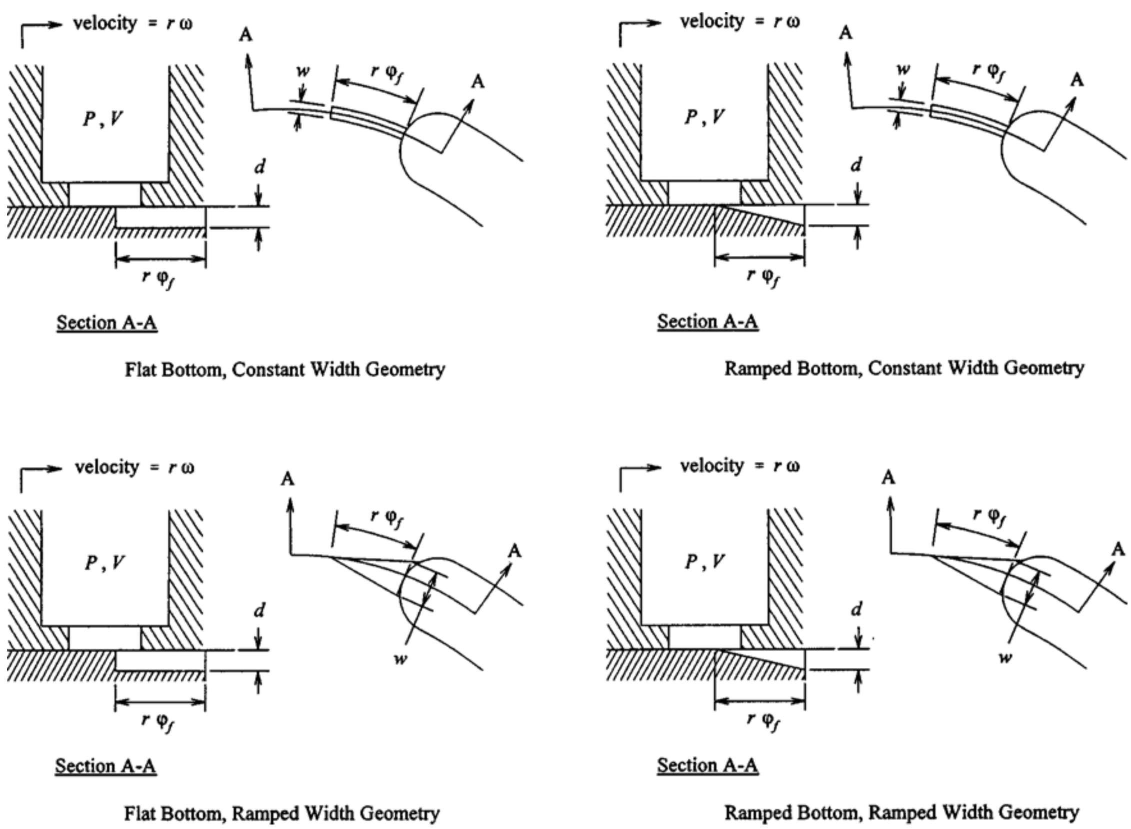

Figure 5: Kidney Port and Cylinder Port Interaction.

From the above equation, the rate of change of pressure depends on the port area variation of the discharge from and to the piston-cylinder chamber. The determination of the port area variation with the angle of rotation is as follows:

Referring to Figures 4 & 5, the port area remains at a maximum constant when it is completely over the discharge or suction manifolds. In the transition region, the port area goes from a maximum to a minimum and then increases. The variation of the orifice area with the angle of rotation is derived below.

The figure- 5 shows the manifold configuration at the Top Dead Center (TDC) and Bottom Dead Center (BDC). At the beginning of the discharge manifold or suction manifolds, a silencing groove is provided to provide the change in flow and pressure experienced by the piston chamber gradually. The end of one manifold and start of another manifold has an angular gap of Ψ'. The angle covered by the orifice area indicated by the kidney pattern in the figure is given by Ψ and the angle covered by the silencing groove is given by φ_f. The silencing groove in this analysis is considered to be a part of the respective manifold to which it is connected. There are various configurations of the silencing grooves as pictured below

In the present analysis, the third configuration, i.e., silencing groove with flat bottom and ramped width geometry is considered.

For simplicity of the analysis, the port areas are linearized. Let the angle covered of the right end of the silencing groove by the barrel kidney port be φ. Also the kidney ports of the barrel is considered to be rectangular and also the opening and closing ends of the actual valves. By simple geometry principles, the following results are obtained. It is assumed that $\Psi > \varphi_f$ and $\Psi = \Psi'$. The area of the groove facing the kidney port is given by,

$$A_g = 0.5(r\psi_f)w \qquad (27)$$

and the area of the kidney port of the barrel is considered to by A_k.

Therefore, the variation of port area of the ith. chamber is given by the following relation:

$$for \ \ 0 \le \varphi \le \varphi_f, \quad A_o = A_g \left(\frac{\varphi}{\varphi_f} \right)^2$$

$$for \quad \varphi_f < \varphi \le \psi, \quad A_{oi} = A_g + A_k \left(\frac{\varphi - \varphi_f}{\psi} \right)$$

$$for \quad \psi < \varphi \leq (\psi + \varphi_f), \quad A_{oi} = A_g \left(\frac{\varphi - \psi}{\varphi_f} \right)^2 + A_k \left(\frac{\varphi - \varphi_f}{\psi} \right)$$

$$\left. for \quad (\psi + \varphi_f) < \varphi \leq \varphi_p, \quad A_{oi} = A_k \right\} \tag{28}$$

$$for \quad \varphi_p < \varphi \leq (\varphi_p + \psi), \quad A_{oi} = A_k \left(\frac{\psi - (\varphi - \varphi_p)}{\psi} \right)$$

The above area variations are used to analyze the pressure variations in each of the pistons.

Numerical Example:

For obtaining the results for various variables of both ideal and real cases, MATLAB framework is used. (Matlab Programming Source Codes are given in appendix- Zap-M05L18).

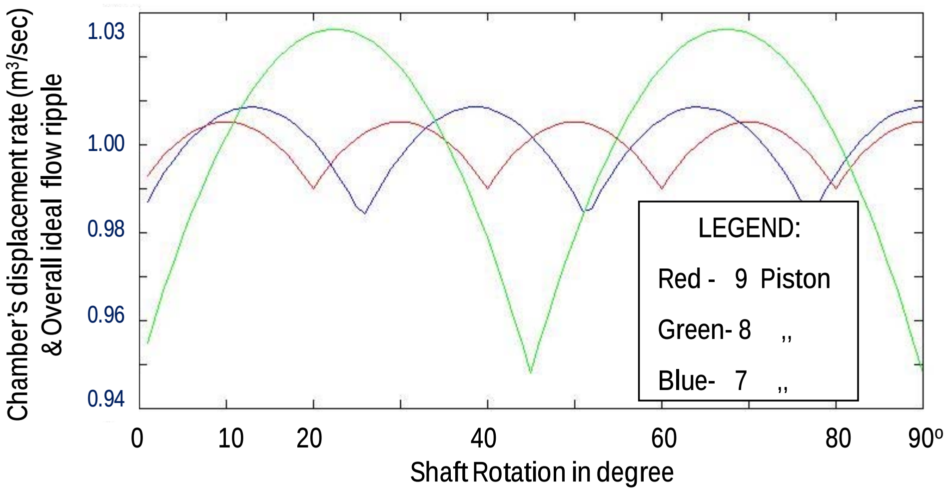

Fig. – 6: Overall Flow Ripple.

Ideal flow rate:

Fig.-6 shows the ideal flow rate with the pump's shaft rotation. It can be seen that from the analysis of the ideal case flow, the pump with even number of pistons has larger pulse time period and also larger amplitude than the odd number counterpart (as we have discussed earlier with phasor analysis). Such ripples are undesirable due to the resulting large fluctuations and noise.

A case study:

For the real case analysis, the groove area variation with the angle of rotation is required is graphical representation of that is given by the following:

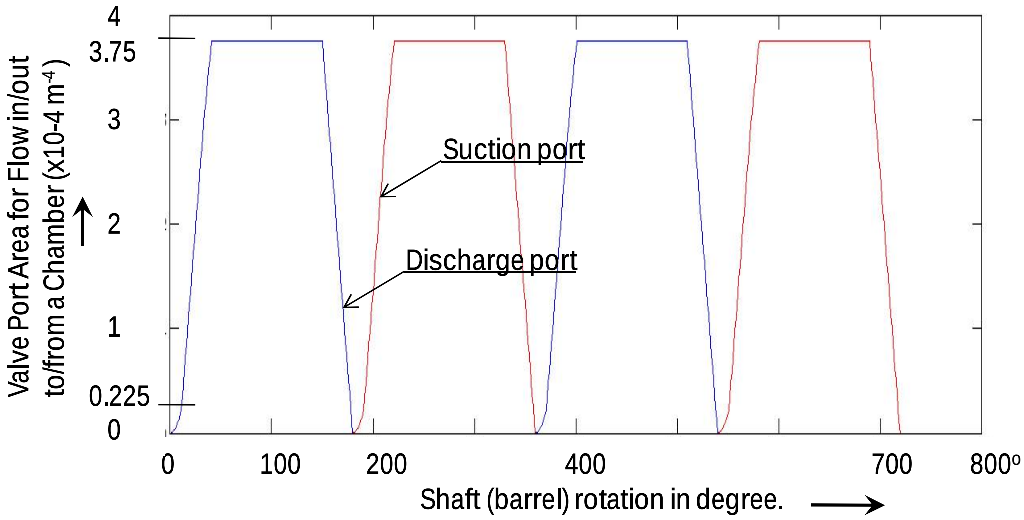

Fig.- 7: Variation in port area A_o with shaft rotation.

SYMBOL	DESCRIPTION	VALUE(SI UNITS)
β	Bulk modulus of the fluid	$10^9 Pa$
C_d	Discharge coefficient of the orifice	0.62
ψ	Angular extent of the barrel orifice (one chamber)	30^o
ψ_f	Angular extent of each silencing groove	11^o
A_g	Maximum opening area of the groove	$2.25\times10^{-5} m^2$
A_k	Maximum area of each barrel port	$3.75\times10^{-4} m^2$
K	Leakage coefficient	$10^{-12} m^3 / Pa$
α	Swash plate inclination with the vertical axis	$0.314\ rad\ (18^o)$
V_0	Nominal volume of a single piston chamber	$22.85\times10^{-6} m^3$
r	Pitch radius of Piston spacing on barrel	$5.501\times10^{-2} m$
ω	Rotational speed of the barrel/block	$235\ rad / \sec$
ρ	Density of the fluid	$850\ kg / m^3$

Table 1: Typical Specifications of a Pump (With 7, 8 & 9 Pistons).

Fig. 7 presents the valve area opening A_o at shaft rotational angle change. The blue graph indicates that the port is over discharge manifold and red indicates that port is over the suction manifold.

The discharge pressure and suction pressures are considered to be p_d = 10 MPa and p_s = 2 MPa with an initial pressure in the piston as 0. The other various parameters used from the analyses are shown in Table- 1.

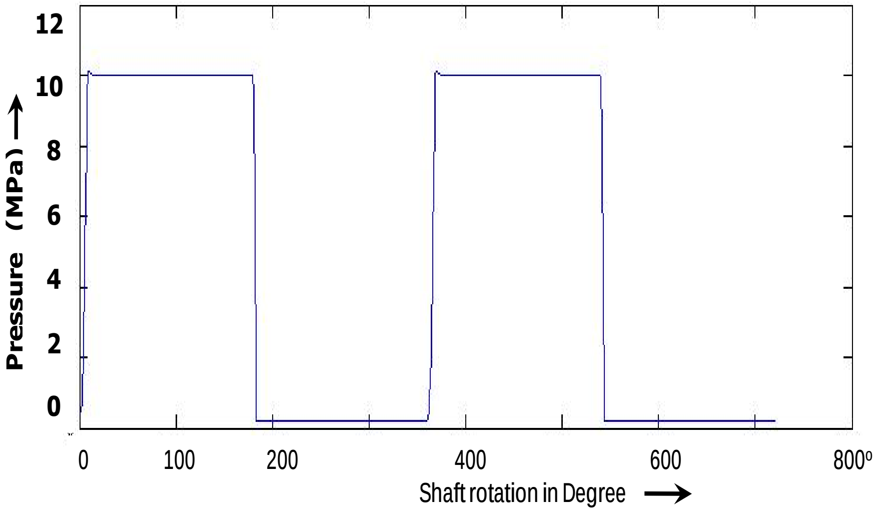

Fig.- 8: Pressure build up in pumps [Ref Data table-1].

Fig. 8 shows the pressure history in a pump. It can be seen that during the transition of the port area from discharge and suction manifolds, the pressure falls rapidly and linearly to the suction manifold's pressure and vice versa, which is presented in Fig. 9.

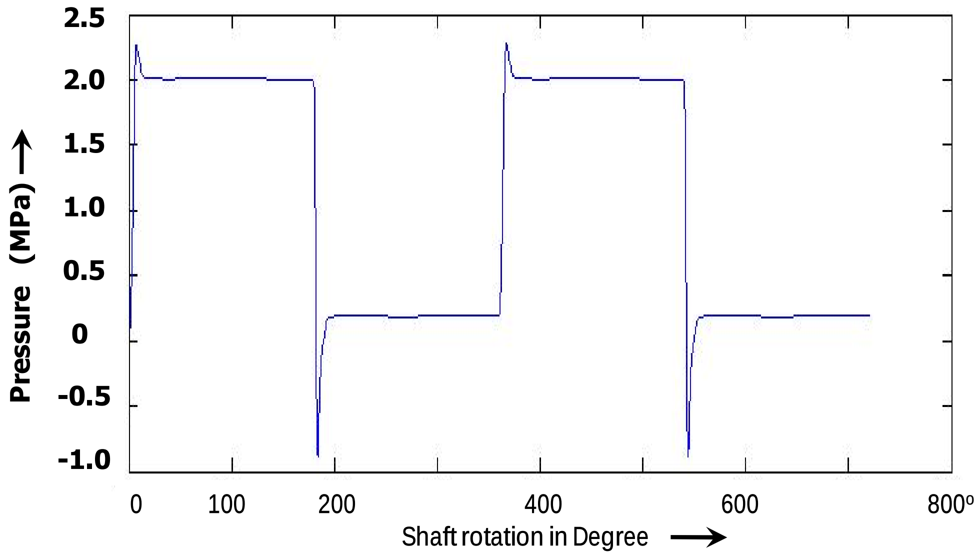

Fig.- 9: Pressure Buildup in a Piston Considering Transient at Valve Entry.

The pressure spikes on the suction side during the transition from discharge to suction manifolds and on the discharge side during the transition from suction to discharge manifolds are observed. The reason for such spikes is that during the transition from discharge and suction manifolds, the chamber volume decreases at a rate faster than the fluid can squeeze out through the port. If the boundary pressure is not sufficiently large compared to the starting pressure, volumetric compression of the fluid will cause the pressure within the piston bore to overshoot the approaching boundary condition. Major attentions are put into the valve design for designing such pumps. Fig.- 9 shows simulated overall flow ripples for pumps with for 7 pistons (blue), 8 pistons (red) and 9 pistons (green).

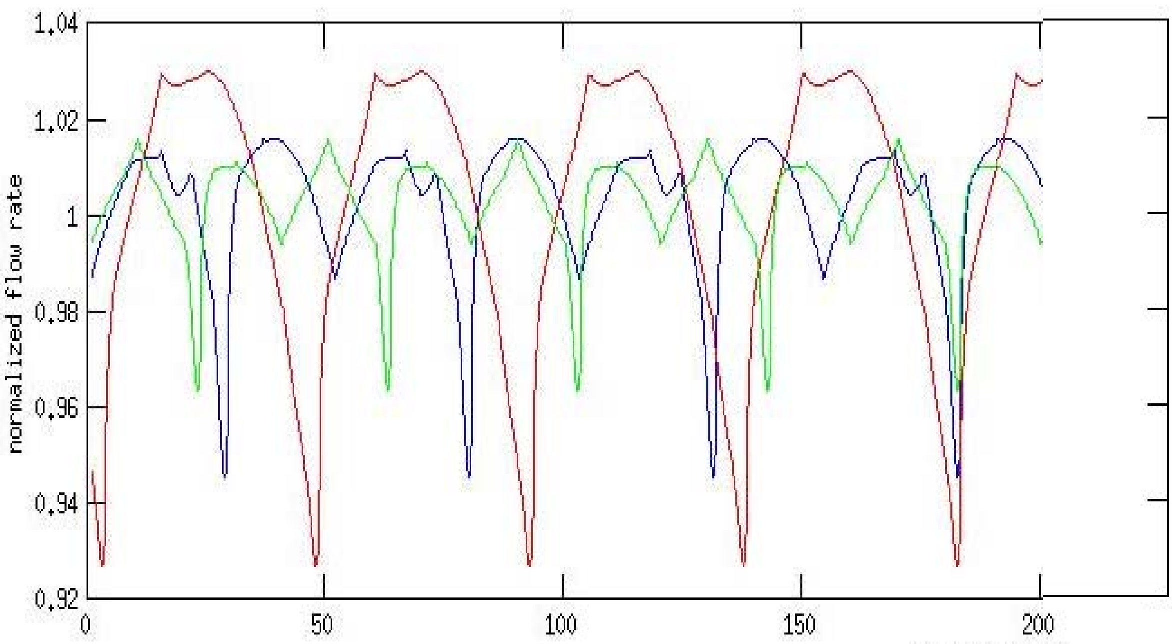

Fig.- 9: Simulated Overall Flow Ripple.

Summary

From the idealized results, one can generally conclude that the flow ripple produced by even number of pistons is significantly larger than a pump with an odd number of pistons. On the other hand, the graph of the actual flow indicates that the difference in the flow ripple characteristics between the even and odd (consecutive) numbered piston pump designs reduces.

It is also to be noted that the frequency and amplitude of each pulse shape are similar irrespective of the number of pistons.

While the idealized flow analysis did not consider the compressibility of the fluid, the numerical investigations results showed that considerable flow variations occur due to the instantaneous compressibility effects that are present during the piston transition as it moves from the intake port to the discharge port.

It can be concluded that there are significant differences between the idealized analysis that has been traditionally used to predict the flow ripple of the pump and numerical analysis which considers the compressibility of the fluid. Thus it may be concluded that it may be feasible to use an even numbered piston design from a flow ripple point of view.

Note: See appendix- ***A_M05_LN_19*** for Matlab Programming Source Codes.

References:

1. Manring, N. D. 2000. The discharge flow ripple of an axial-piston swash-plate type hydrostatic pump. **ASME *Journal of Dynamic Systems, Measurement, and Control*.** 122:263-268.
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3. Mandal, N, P. , Saha R. and Sanyal D. 2008. Theoretical simulation of ripples for different leading- side groove volumes on manifolds in fixed-displacement axial-piston pump. **IMechE (UK) J. Systems and Control Engineering.** (Part-I) 222:557-569.