Module- 4: ELECTRO-HYDRAULICS VALVES

LECTURE- 14: Critical Center Spool Valve Analysis-Stroking force

Stroking forces

Force required to drive a spool at an instant is the instantaneous stroking force. It has usually steady state, transient and dynamic components. Not only at the vicinity of control point but also during spool motion from a position to other position the flow force (steady state part) dominates.

Flow forces & static pressure forces

1) ***Flow forces on spool valves***

It is named as flow induced forces, Bernoulli forces or hydraulic reaction forces. The force induced as a result of flows through the valve orifices and valve body passages.

Referring to Fig. 4.14-1, the steady-state flow forces (jet), F_j can be expressed as,

$$F_j = \rho V \frac{Q_2^2}{A_2 V} = \frac{\rho Q_2^2}{A_2} = \frac{\rho Q_2^2}{C_c A_0} \qquad \ldots 4.14\text{-}1$$

(Mass, Accn.)

Where, $Q = V$olumetric flow rate through orifice, m^3 / sec

$V = C$ontrol volume being accelarated, m^3

$A_0 = wx_v = O$rifice area, m^2

$C_c =$ Contraction coefficient, (dimensionless)

$C_v =$ Velocity coefficient, (dimensionless)

$C_d = C_v C_c =$ Coefficient of discharge, (dimensionless)

$\rho =$ Oil mass density, kg / m^3

$w =$ Area gradient of orifice

$(= \pi \times$ spool land dia. at orifice), m^2 / m

$A_v =$ Area (annular) of valve land, m^2

$\theta =$ Jet angle, degrees.

Force F_j is now resolved in their reaction force components (Newton's third law). The axial component,

$$F_1 = -F_j \cos\theta \qquad \ldots 4.14\text{-}2$$

and the lateral component,

$$F_2 = -F_j \sin\theta \qquad \ldots 4.14\text{-}3$$

Lateral force causes sticking the spool to sleeve or valve body bore wall. It is balanced by symmetrical porting.

Continuity requires that,

$$Q_1 = Q_2 \qquad \ldots 4.14\text{-}4$$

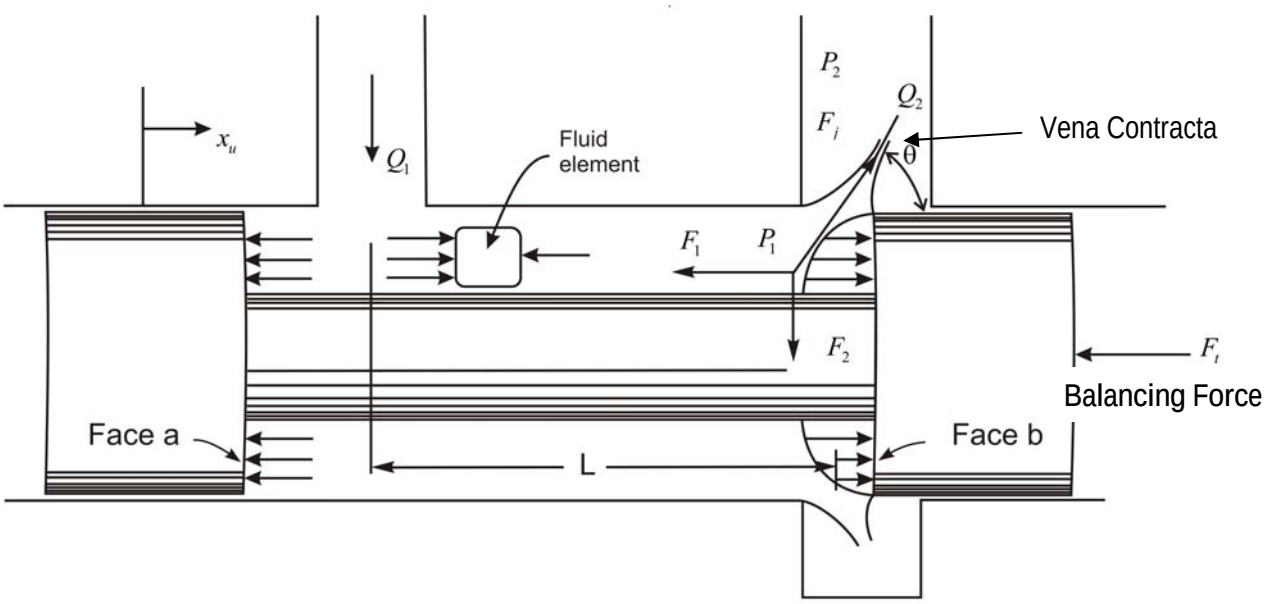


Figure 5-11 Flow forces on a spool valve due to flow leaving a valve chamber.

Therefore, using orifice equation,

$$Q_1 = Q_2 = C_d\, A_0 \left[\frac{2}{\rho}(P_1 - P_2)\right]^{1/2} = C_c\, C_v A_0 \left[\frac{2}{\rho}(P_1 - P_2)\right]^{1/2} \qquad \ldots 4.14\text{-}5$$

Substituting in eqn. 4.14-1,

$$F_1 = \frac{-\rho\left(C_c\, C_u A_0\right)^2 \left[\frac{2}{\rho}(P_1 - P_2)\right]}{C_c\, A_0} \cos\theta$$

Therefore, F_1 along the shown direction is finally,

$$F_1 = 2 C_d\, C_u\, A_0\, (P_1 - P_2) \cos\theta \qquad \ldots 4.14\text{-}6$$

This force will try to close the port.

If the orifice is rectangular and the peripheral width is large in comparison to axial opening, then the flow is usually two dimensional and may be assumed as irrotational, nonviscous and incompressible. Von Mises used LaPlace's equation to find jet angle θ. For all practical purposes with very insignificant radial clearance θ may be taken as 69^o (see Fig.- 4.14-2).

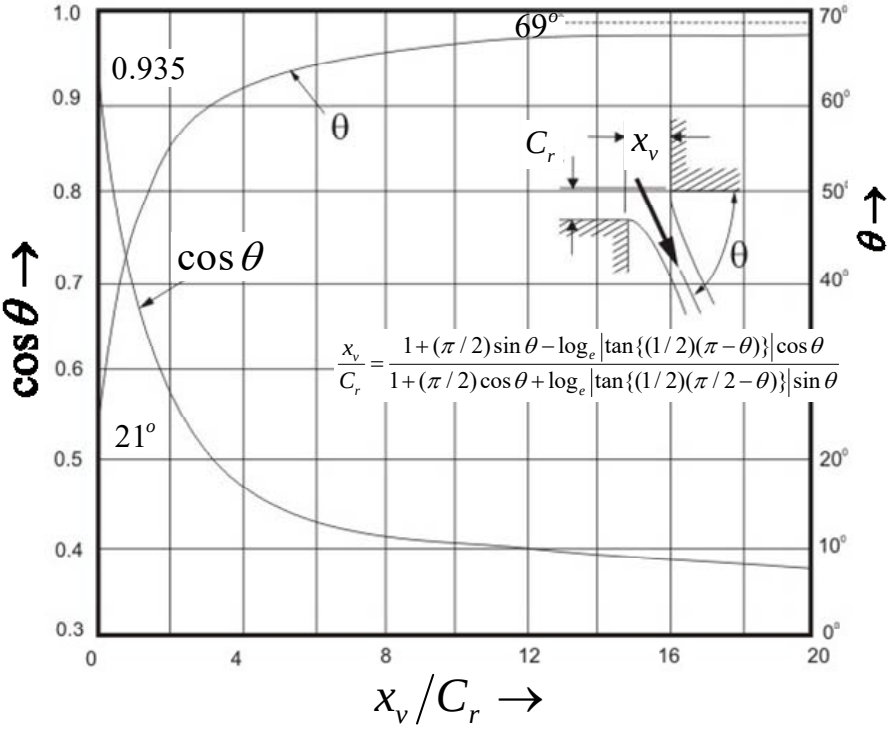


Fig. 4.14-2: Effect of radial clearance on the jet angle

At small orifice openings i.e. at the beginning of opening θ remains close to 21^o (See Fig. 4.14-2). The steady state force then expressed as,

$$F_1 = 2\,C_d\,C_v\,w(\Delta P)\sqrt{x_v^2 + C_r^2}\cos\theta \qquad \ldots 4.14\text{-}7$$

However, experimental results show higher values than estimated values and stiffer curves near the null point, by using above formulae.

Transient flow forces

Transient flow force F_3 due to fluid mass is expressed as:

$$F_3 = Ma = \rho L\ A_v\ \frac{d\left(Q_1 / A_v\right)}{dt} = \frac{\rho L dQ_1}{dt} \qquad \ldots 4.14\text{-}8$$

L is called damping length & A_v is the cross section inside the spool chamber.

Obtaining dQ_1 / dt from 4.14-5, flow force at transient zone is expressed as,

$$F_3 = LC_d w\sqrt{(2/\rho)(P_1 - P_2)}\,\frac{dx_v}{dt} + \frac{LC_d w x_v}{\sqrt{(2\rho)(P_1 - P_2)}}\,\frac{d(P_1 - P_2)}{dt} \qquad \ldots 4.14\text{-}9$$

Clearly, transient flow force is proportional to spool velocity and change in pressure drop. The velocity term, which represents damping force, is more sensitive. The pressure part has negligibly small contribution towards the valve dynamics. It is to be noted that for reverse flow i.e., flow entering through the orifice (right end orifice in Fig. 4.14-1) the transient flow force is destabilizing. Care must be taken in analysis of force balancing as there are few ports in series along the spool.

Stroking force

Referring to the Fig-4.12-1 let x_v is positive for orifice 1 & 3 the total force opposing the spool motion is expressed as,

$$\begin{aligned} F_R &= 2C_d\, C_v(\cos\theta)\, w x_v (P_s - P_1) - L_1 \rho \frac{dQ_1}{dt} \\ &\quad + 2C_d\, C_v(\cos\theta)\, w x_v P_2 + L_2 \rho \frac{dQ_3}{dt} \end{aligned} \qquad \ldots 4.14\text{-}10$$

For symmetrical and matched valve,

$$F_R = 2C_d\, C_v(\cos\theta)\, w(P_s - P_1)\, x_v + (L_2 - L_1) C_d w\sqrt{\rho(P_s - P_1)}\ \frac{dx_v}{dt} \qquad \ldots 4.14\text{-}11$$

The pressure derivative part $\frac{dP_L}{dt}$ terms are negligibly small and not considered.
Now the equation of motion of the spool valve is as follows,

$$F_i = M_s \frac{d^2 x_v}{dt^2} + B_f \frac{dx_v}{dt} + K_f x_v \qquad \ldots 4.14\text{-}12$$

Where, M_s = Mass of the spool.

The damping coefficient B_f is expressed as,

$$B_f = (L_2 - L_1) C_d w \sqrt{\rho (P_s - P_1)} \qquad \ldots 4.14\text{-}13$$

and the spring coefficient (spring rate) K_f is expressed as,

$$K_f = 2 C_d C_v (\cos\theta) w (P_s - P_1) \qquad \ldots 4.14\text{-}14$$

Analyses show that,
The steady state flow force acts as centering spring on the valve.
The transient flow force acts as viscous damping.
Both quantities are non linear because of changes in P_L.
If $L_2 < L_1$, the transient flow force is negative and may cause valve instability.

Therefore, in design it is maintained that $L_2 \geq L_1$

Now the "damping ratio" may be defined as

$$\delta_s = \frac{B_f}{2\sqrt{K_f M_s}} \qquad \ldots 4.14\text{-}15$$

Substituting the expression of B_f and K_f we get,

$$\delta_s = \frac{(L_2 - L_1) C_d w \sqrt{\rho (P_S - P_L)}}{2\sqrt{2 C_d C_v w (\cos\theta)(P_S - P_L) \times M_s}} \qquad \ldots 4.14\text{-}16$$

$$= \frac{(L_2 - L_1)}{2\sqrt{2\cos\theta}} \sqrt{\frac{C_d \rho w}{C_v M_s}}$$

Finally,

$$\delta_s = (L_2 - L_1) \left\{ \frac{1}{2} \frac{C_d \sqrt{\rho}}{\sqrt{2 C_v C_d \cos\theta}} \right\} \sqrt{\frac{w}{M_s}} \qquad \ldots 4.14\text{-}17$$

It is independent of pressure $\left[(P_S - P_L) \right]$

Practical design

In a practical design,

$$L \approx 6d \qquad \ldots 4.14\text{-}18$$

Where, d is spool land diameter (See Fig. 4.12-1).

And the damping length

$$L_1 + L_2 \approx \frac{L}{3} = 2d \qquad \ldots 4.14\text{-}19$$

Also, the spool rod diameter may be taken as,

$$d_r \approx \frac{d}{2} \qquad \ldots 4.14\text{-}20$$

The spool volume is also approximated as,

$$\frac{\pi}{4}d^2L - 2\left(\frac{\pi}{4}d^2 - \frac{\pi}{4}d_r^2\right)(L_1 + L_2) = \left(\frac{1}{2}\right)\frac{\pi}{4}d^2L = \frac{3\pi}{4}d^3 \qquad \ldots 4.14\text{-}21$$

$$\therefore M_s = \left(\frac{3\pi}{4}d^3\right)\rho_s \qquad \ldots 4.14\text{-}22$$

Significance of the transient flow force coefficient B_f by computing the damping ratio.

Substituting the expression of M_s in equation 4.14-17, the relation of damping ratio can be rewritten as,

$$\delta_s = (L_2 - L_1)\left\{\frac{1}{2}\frac{C_d\sqrt{\rho}}{\sqrt{2C_vC_d\cos\theta}}\right\}\sqrt{\frac{w}{\left(\frac{3\pi}{4}d^3\right)\rho_s}}$$

$$= (L_2 - L_1)\left\{\frac{1}{2}\frac{C_d\sqrt{\rho}}{\sqrt{2C_vC_d\cos\theta}}\right\}\frac{2}{d\sqrt{3\rho_s}}\sqrt{\frac{w}{\pi d}}$$

$$= \frac{(L_2 - L_1)}{2d}\left\{\frac{C_d\sqrt{\rho}}{\sqrt{2C_vC_d\cos\theta}}\frac{2}{\sqrt{3\rho_s}}\right\}\sqrt{\frac{w}{\pi d}}$$

Considering damping length (eqn. 4.14-19) finally,

$$\delta_s = \frac{(L_2 - L_1)}{(L_2 + L_1)} \left\{ \frac{C_d}{\sqrt{2C_v C_d \cos\theta}} \sqrt{\frac{4}{3}\frac{\rho}{\rho_s}} \right\} \sqrt{\frac{w}{\pi d}} \qquad \ldots 4.14\text{-}23$$

Calculations:

With a good approximation it may be assumed that (re: Fig.-4.14_2),

$\theta \approx 69^0, \quad \cos 69^0 = 0.358$

Also,

$C_d = 0.61, \quad C_v = 0.98.$

both C_d and C_v are dimensionless.

Therefore,

$2C_d C_v (\cos\theta) \approx 0.43$

For steel spool,

$\rho_s = 7.8\, gm/cm^3 = 7.8 \times 10^3\, kg/m^3$

And for hydraulic oil.

$\rho = 830\ kg/m^3$

$$\frac{C_d}{\sqrt{2C_v C_d \cos\theta}} \sqrt{\frac{4}{3}\frac{\rho}{\rho_s}} = \frac{0.61}{\sqrt{0.43}} \sqrt{\frac{4}{3} \times \frac{830}{7800}} = 0.93 \times \sqrt{0.142} = 0.93 \times 0.377 = 0.35$$

Therefore,

$$\delta_s = 0.35 \frac{(L_2 - L_1)}{(L_2 + L_1)} \sqrt{\frac{w}{\pi d}} \qquad \ldots 4.14\text{-}24$$

It is dimensionless and independent of any unit system.

For a rectangular port, which is very common in spool valve, $w = \pi d$. Therefore, $\sqrt{\frac{w}{\pi d}}$ is unity.

In such condition if damping length is 50% greater than thewother (say $L_2 = 1.5L_1$), the damping ratio δ_s becomes 0.07. On the other hand if $L_1 = 1.5L_2$ then δ_s becomes negative with same value.

It would create same effect but with negative value . i.e., magnitude of δ_s can never be greater than $0.35\sqrt{w/\pi d}$. However, a realistic value is around $0.1\sqrt{w/\pi d}$. In practice

$$L_2 \cong L_1 \text{[but with positive damping]}$$

Both viscous damping coefficient and spring rate maximum at null point, at $P_L = 0$.

$$K_{fo} = 0.43 w P_s$$

Mass of the spool.

$$\therefore M_s = (1.84 \times 10^4) d^3 \ kg$$

Where d is in meter.

Example for d=12.5 mm at 7MPa

$$K_{fo} = 0.43\pi 12.5 \times 10^{-3} \times 7 \times 10^6$$

$$= 118.14 \ kN/m$$

Therefore, for a stroke of 0.5 mm at null point spool force required approximately

$$(0.5 \times 10^{-3}) \times (118.14 \times 10^3) \cong 59 \ N$$

Bibliography:

1. Herbert E. Merritt, 'Hydraulic Control System', John Wiley & Sons, Inc., USA, 1967.