Module- 2 : Fundamentals of Fluid Flow and Properties

Lecture - 6 : Basis for Calculating Hydraulic Systems

To calculate a hydraulic system, it is normally necessary to determine the following four quantities.

i) Quantity of energy in the system'
ii) Total pressure drop in the system,
iii) Total leakage drop in the system, and
iv) Total heat development in the system.

In what follows these four quantities will be determined to the considerably simplified example below.

Example of a Hydraulic System

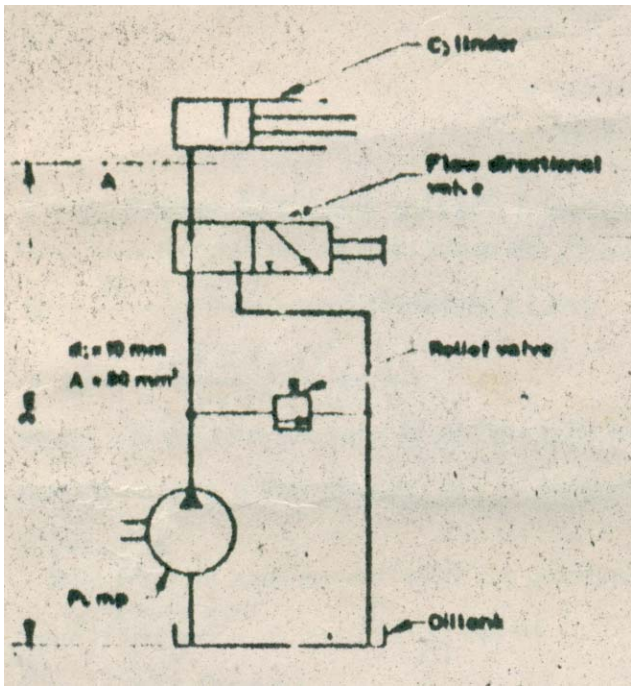


Fig.- 1: Basic Hydraulic System

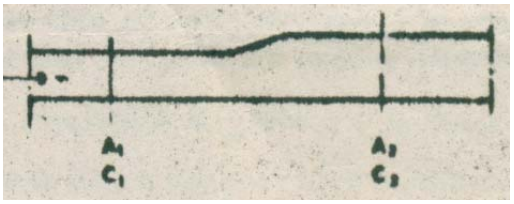


Fig.-2:

Fixed values: (ref. Fig.-1)

$p = 10^6$ kp/m^2 = 100 kp/cm^2
$Q = 5 \times 10^{-4}$ m^3/s 30 l/min
$\rho = 90$ kp $\times$ s^2 $\times$ m^{-4}
$\eta = 34 \times 10^{-4}$ kp $\times$ s $\times$ m^{-2} at 50 ^{o}C (122 ^{o}F)
Oil velocity c = Q/A = 64 m/s

Energy in the Oil System (Ref Fig.-2)

In the oil flows through a tube without any losses, the energy can be determined in an arbitrary cross section, since Bernoulli's equation says.

$$E = \frac{p}{\rho} + \frac{c^2}{2} + gh = \text{constant}$$

In reality there are no liquids which are not subject to losses, and therefore another term has to be added:

$E = \frac{p}{\rho} + \frac{c^2}{2} + gh + \text{loss} = \text{constant}$. In the theoretical, technical system the dimension E is kpm per unit mass = $m^2 \times s^{-2}$. In this conjunction "loss" always means development of heat the first three terms of the equation are studied, it is seen that the energy in a liquid originates partly from the static pressure ($\frac{p}{\rho}$) of the oil, partly from the oil velocity ($\frac{c^2}{2}$), and partly from the geometric height (gh).

Within hydraulics there are partly hydrostatic and partly hydrokinetic system.

The hydrokinetic systems in which much of the energy originates from the oil velocity will not be dealt with here. The two midmost terms ($\frac{c^2}{2}$ and gh) will normally be insignificant relative to the first term . $\frac{p}{\rho}$ in a hydrostatic system.

The following formula is suitable for rapidly determining the quantity of energy in an oil flow whose the capacity is Q l/min. and the pressure is Δp kp/cm^2.

$N = \frac{Q \times p}{\pi T}$.(Note: Not the theoretical system)

To calculate the velocity in different cross sections of the system the equation of continuity is used which with good approximation ($\rho_1 = \rho_2$) can be written as follows:

$$A_1 c_1 = A_2 c_2 \ .$$

Calculating the Energy in the Example.

The heat content being disregarded, the energy per unit mass at the cross section A - A is as follows:

$E_M = \frac{p}{\rho} + \frac{c^2}{2} + gh =$ m^2/s^2 ;

$E_M = \frac{100 \times 10^4}{90} + \frac{6.4}{2} + 9.81 \times 3$ m^2/s^2

$E_M \approx 11111+20+29=11160$ m^2/s^2

The energy per second is:

$$N = E_M \times Q \times \rho;$$

$N \approx 11160 \times 5 \times 10 \times 90 = 502$ kpm/s.

The other formula gives N directly:

$$N = \frac{p \times Q}{6} = \frac{100 \times 30}{6} = 500 \text{ kpm/s.}$$

Pressure Drop in the Oil System.

When oil is flowing through a tube, the pressure drops in the direction of flow. This pressure drop always depends on the oil velocity, tube length, and tube diameter. Furthermore, any alteration of the velocity will result in substantial pressure drops. Such alterations of velocity occur in, for example, tube bends, valves, and with any alternations of the cross section.

As the size of the pressure drop varies much according to whether the flow is laminar (often called viscous) or turbulent, it is necessary first to determine the kind of flow.

To this end an undimensioned quantity is used which is

denoted by Reynolds numbers "Re"

Re is fixed at $\frac{c \times d}{\nu}$ where c is the oil velocity in m/s d is the diameter in m, and ,ν is the viscosity in m^2 /s.

If for round pain tubes Re exceeds 2300, the flow is said in be turbulent. It Re is less than 1200, the flow is called laminar. The interval is a transition area which should rather be avoided when the tube dimensions for systems of a more complicated nature are to be determined.

If the liquid flows through slots or similar openings, the

transition from laminar to turbulent flow takes place at a lower

Re number.

For leakage flow in pumps and motors the most recent experience shows further that in spite of every low Re number the flow will often be a mixture of turbulent and laminar flows. To calculate pressure drops and leakage losses in such cases it is necessary to calculate it a losses for each kind of flow separately and add them afterwards.

Pressure Drop with Laminar Flow.

With flow through round plain tubes the pressure drop is:

$$\Delta p = \frac{32 \times c \times \eta \times l}{d^2} \text{ kp/m}^2 \text{ (Poissulle's Law)}$$

Or, $\Delta p = \frac{128 \times Q \times \eta \times l}{\pi \times d^2}$ kp/m^2

Where d is the tube diameter in m, l the tube length in m, c

the velocity in m/s, η the viscosity in kp × s × m^{-2}, and $Q = \frac{\pi}{4} d^2 c$ m^3/s.

With flow through annular slots the pressure drop is:

$$\Delta p = \frac{12 \times c \times \eta \times l}{\delta^2} \times \frac{l}{\left(l + 1.5 \times \varepsilon^2\right)} \quad \text{kp/m}^2$$

Or, $\Delta p = \frac{12 \times Q \times \eta \times l}{\pi D \delta^2} \times \frac{l}{\left(l + 1.5 \times \varepsilon^2\right)}$ kp/m^2

Dimensions as specified above, however δ = mean slot height in m, D = mean diameter in m, ε is eccentricity, and $Q = \pi D \delta c$ m^3/s .

It should be noted that for ε=1 the pressure drop is 2.6 times as small as for ε=0 for the same oil volume.

With flow through a plane slot the pressure drop is:

$$\Delta p = \frac{12 \times c \times \eta \times l}{\delta^2} \quad \text{kp/m}^2$$

Or, $\Delta p = \frac{12 \times Q \times \eta \times l}{b \delta}$ kp/m^2

Dimensions as specified above, however b = slot width and
$Q = b \delta c$ m^3/s.
it the undimensioned quantity l is inserted in Poissulle,s

Formuls where $\lambda = \frac{64}{\text{Re}} = \frac{64 \times \gamma}{c \times d} = \frac{64 \times \eta}{c \times d \times \rho}$ the result is the following equation for round tubes:

$$\Delta \rho = \lambda \times \frac{l}{d} \times \frac{c^2}{2} \times \rho \quad \text{kp/m}^2 .$$

When divided by the specific gravity $\gamma = \rho$ g. the rusult is:

$h_f = \lambda \times \frac{l}{d} \times \frac{c^2}{2g}$ "m" (Dercy's Formula).

Dimensions as specified above and $h_f = \frac{\Delta p}{\gamma}$ ----

γ (often replaced by "f ") can be determined when the Re number has been calculated.

Pressure Drops with Turbulent Flow.

To calculate the pressure drop with turbulent flow through a tube (does not apply to nozzles), the empirical formula is so be used:

$$\Delta p = \lambda \times \frac{l}{d} \times \frac{c^2}{2} \times \rho \quad \text{kp/m}^2 .$$

Or, $\Delta p = \frac{8}{\pi^2} \times \lambda \times \frac{Q^2 \times \rho \times l}{d^2}$ kp/m^2

Dimensions as specified above and $Q = \frac{\pi}{4} d^2 c$ m^3/s.

As is soon the pressure drop can be expressed in the same way whether it is a question of laminar or turbulent flow. The tube friction coefficient λ is, however, very different in the two instances which results in the previously mentioned difference in pressure drop (see below).

In the case of turbulent flow λ depends partly on the number and partly on the relative roughness in the tube. The relative roughness is Δ/d where d is the diameter and Δ is the roughness which depends on the tube quality.

When the Re number and the relative roughness have been calculated, λ is found in a table or by means of a curve for λ as a function of Re and with the relative roughness as parameter.

If mean roughness is used, λ can be fixed at $0.3164 \times (Re)^{-1/4}$, It should be noted that the pressure drop with laminar flow is $\Delta p = k_1 \times c$ according to the above while the pressure drop with turbulent flow is $\Delta p \approx k_2 \times c^2$ (this result is had by fixing λ at $\lambda = k_3 \times c^{-1}$ and $\lambda = k_4 \times c^{-1/4}$ respectively.

If the tube is not round, the "hydraulic radius" "r_p " is used instead of the diameter "d".

$$r_p = \frac{\text{Cross-sectional area of the tube}}{\text{Circumference of tube}}$$

For a round cross section r_p is thus d/4.

The pressure drop for a tube with length l and an arbitrary cross section with hydraulic radius "r_p " is thus.

$$\Delta\rho = \lambda \times \frac{l}{4 \times r_p} \times \frac{c^2}{2} \times \rho \text{ kp/m}^2 .$$

Dimensions as specified above.

It should be noted that this pressure drop is only the drop in the tube, while the pressure drops at the tube ends are determined in another way (see below).

If the tube is very short (d = l), the total pressure drop can be determined by means of the nozzle formula.

$$Q = C_d \times A \times \sqrt{\frac{2 \times \Delta p}{\rho}} \text{ m}^3/\text{s}$$

where $\Delta p = \frac{1}{2} \times \frac{Q^2 \times \rho}{C_d^2 \times A^2}$ kp/m^2

Dimensions as specified above. The nozzle coefficient "C_d" depends on the admission edge but is independent of the shape and size of the nozzle cross section. In this instance, C_d can be fixed with good approximation at 0.66.

As previously mentioned, bends, branching-offs expansion, narrowings and valves etc. will also cause pressure drops to occur. These pressure drops cannot be calculated on the basis of the formulae already mentioned.

Therefore, another non-dimensional term has been introduced.

$$\xi = \lambda \times \frac{l}{d}$$

The pressure drop in a tube can then also be

written as $\Delta p = \xi \times \frac{c^2}{2} \times \rho$ kp/m .

Dimensions as specified above.

This formula applies to both turbulent and laminar flow in tubes irrespective of the shape of the tube cross section. In addition, the formula can be used to calculate pressure drops in bends, branching offs, and valves, etc. Only ξ should be found first in every single instance for round tubes. ξ can be calculated by means of the above formula, but in all other instances ξ must be taken from tables in books or from catalogues since in that case ξ is a quantity which has been determined through experiments. When selecting ξ it is necessary to know whether the flow is laminar or turbulent, since this results in the different values. Normally, turbulent, flow is ..... with ..... ξ is stated.

If the ξ-value for a given valve is known, the "equivalent tube length" $l_{equiv.}$ of the valve can be calculated by means of the formula :

$$l_{equiv} = \xi \times \frac{d}{\lambda}$$

The equivalent tube length for tube bends, etc. can be determined in the same way.

Leakage Loss in the Oil System.

The formulae of pressure drops in the preceding section immediately give the oil volume per second with laminar as wall as turbulent flow.

In pumps, motors, cylinders, and valves etc. part of the oil volume will always flow from the delivery side to the return side through the slots which will always exist between the moving parts.

If the flow is laminar, this leakage flow is:

$Q_l = \frac{\pi \times d^4 \times \Delta p}{128 \times \eta \times l}$ m^3/s for a slot with round cross section.

$$Q_l = \frac{\pi \times D \times \delta^3 \times \Delta p}{12 \times \eta \times l} \times \left(1 + 1.5 \times \varepsilon^2\right) \text{ m}^3/\text{s}$$ for an annular

slot with eccentricity ε, and

$$Q_l = \frac{b \times \delta^3 \times \Delta p}{12 \times \eta \times l} \text{ m}^3/\text{s}$$ for a plane slot.

Dimensions as specified in the previous section. If the flow is turbulent, the oil flow is:

$$Q_t = C_d \times A \times \sqrt{\frac{2 \times \Delta p}{\rho}} \text{ m}^3/\text{s}$$ since the slot length is always

rather short so that the resistance in the slot proper is so small relative to the resistance at the ends that the nozzle formula can be used.

As previously mentioned, both formulae of leakage flow will in some cases be used together so that the total leakage flow is $Q_1 + Q_t$.

Example of Calculating Pressure Drops.

Calculating the pressure drop between the pump and valve in the previous example 1 = 2 m., c = 6.4 m/s ;

$$\nu = \frac{\eta}{\rho} = \frac{34 \times 10^{-4}}{90} = 38 \times 10^{-6} \text{ m}^2/\text{s}.$$

First determine the Re number:

$$\text{Re } \frac{c \times d}{\nu} = \frac{6.4 \times 10 \times 10^{-3}}{38 \times 10^{-6}} = 1700$$

If the flow is reckoned as being laminar, the pressure droop is: -4

$$\Delta p = \frac{32 \times c \times \eta \times l}{d^2} = \frac{32 \times 6.4 \times 34 \times 10^{-4} \times 2}{\left(10^{-2}\right)^2} \approx 14000 \text{ kp/m}^2 .$$

$\Delta p = 1.4$ kp/cm^2 .

If the flow is reckoned as being turbulent, the pressure drop is:

$$\Delta p = \lambda \times \frac{l}{d} \times \frac{c^2}{2} \times \rho = \frac{0.3164}{.....} \times \frac{2}{0.01} \times \frac{6.4^2}{2} \times 90 \text{ kp/m}^2$$

$\Delta p = 18000$ kp/m^2 $= 1.8$ kp/cm^2 .

Example of Calculating Leakage Loss.

Calculating the leakage loss across the piston in the cylinder. It is a condition that the piston is without packings thus D = 50 mm., l = 40 mm, δ = 0.01 mm and ε = 0.

$$Q_l = \frac{\pi \times D \times \delta^3 \times \Delta p}{12 \times \eta \times l} \times \left(1 + 1.5 \times \varepsilon^2\right) = \frac{\pi \times 50 \times 10^{-3} \times \left(0.01 \times 10^{-3}\right)^3 \times 100 \times 10^4}{12 \times 12 \times 10^{-4} \times 40 \times 10^{-3}} \times \left(1 + 1.5 \times 0^2\right) \text{ m}^3/\text{s}$$

Q=0.096 x 10^{-6} m^3/s=0.096 cm^3/s

If for example, δ was doubled (=0.02 mm) and ε =1, the leakage would be 20 times as large.

Heat Development in the Oil System

In hydraulic systems development of heat is always taking place which is due to pressure energy being converted into heat. Such conversion of energy takes place where the oil flows through slots from a high pressure towards a lower one. This takes place partly in almost any hydraulic components in the form of leakage flow from the delivery side to the return side (a leakage flow may very well be intentional since it lubricates and cools bearings, etc.) and partly as throttling in valves (regulating valves, overflow valves, etc.)

if Q m^3 /s flows through a slot at a pressure drop of Δp kp/m^2 a quantity of heat is developed:

$$K_q = \frac{Q \times \Delta p}{427} \text{ kcal/s.,}$$

If the oil is not re-circulated, and the increased heat emission to the surroundings is disregarded, the corresponding temperature increase is:

$$\Delta T = \frac{K_q}{c \times Q \times \rho} = \frac{\Delta p}{c \times 427 \times \rho} = \frac{\Delta p}{427 \times 4.3 \times 90} = 6 \times 10^{-6} \times \Delta p \ ^{o}\text{C}$$

In this instance the heat equivalent is fixed at 427 kpm/kcal, and the specific heat is fixed at 4.3 kcal/Te kma × ^{o}C (≈ 0.44 kcal/kg x ^{o}C).

The final temperature in the oil system is determined as the temperature T_s at which the heat emission to the surroundings K_k equals the heat supply K_q .

To calculate the heat emission K_k to the surrounding, the formula below is used.

$K_k = k \times A \times \Delta T$ kcal/s.

Where A = the outer surface in m^2

ΔT = the temperature difference between the oil and the surroundings.

k = heat transfer coefficient per second for metal surfaces (oil tank and tubes) to which the air has free access for natural cooling. k can be fixed at 0.0036 kcal/m^2 × s × ^{o}C.

Normally, a system is calculated in such a way that the final temperature is about 50 ^{o}C (122 ^{o}F), since 60 ^{o}C (140 ^{o}F) is regarded as the maximum permissible oil temperature.

It should be noted that normally it is unfortunate to run oil lines along hot surfaces since sufficient heat emission is thus impeded.

Example of Calculating the Final Temperature.

It is a condition that the cooling surface of the system is 2 m^2 (oil tank and tubes), that the air temperature is 20 ^{o}C (68 ^{o}F), that the leakage oil volume and the oil volume passing through the relief valve constitutes 20% on an average of the total oil volume Q = 5×10^{-4} m^3 /s.

By using the formula of heat balance, the result is :

Equation

$K_q = K_k$, i.e. $\frac{Q \times \Delta p}{427} = k \times A \times \Delta T$

$$\frac{\left(5 \times 10^{-4} \times 1/5\right) \times 100 \times 10^4}{427} = 0.0036 \times 2 \times \left(T_s - 20\right)$$

$\left(T_s - 20\right)^{o}$=32.4 ^{o}C (90 ^{o}F)

T_s = 52.4 ^{o}C (126 ^{o}F).

Reference:

‘Basis for Calculating Hydraulic Systems’, Danish Technical journal, No. 2 and 3, 1962.
By V. Bender M. Sc. (Eng.) Danfoss, Nordborg, Denmark