

Sound Propagation through Media

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LECTURE-27

TRANSMISSION BETWEEN TWO FLUIDS AT OBLIQUE INCIDENCE

Introduction

- In this lecture, we continue to explore propagation of sound waves through two fluid media at oblique incidence angles.
- Till so far, we have developed expressions for propagation vectors, and also for transmission and reflection coefficients.
- Next, we explore how relationships for power transmission and reflection coefficients for obliquely incident wave. We also introduce the concepts of “angle of intromission” and “normal specific acoustic impedance”.

Power Coefficients

- Next we calculate power reflection coefficient (R_p) and power transmission coefficient (T_p).
- Since, the cross-sectional area of incident and reflected wave is same, we can state:

$$R_p = R_I = |R|^2 \quad \text{Eq. 27.1}$$

- Also, from principle of conservation of energy, we can state:

$$T_p = 1 - R_p = 1 - |R|^2 \quad \text{Eq. 27.2}$$

Power Coefficients

- Using Eqs. 27.1 and 27.2, we can write:

$$R_P = \begin{cases} \frac{(Z_2/Z_1) - \cos \bar{\theta}_T / \cos \theta_I}{(Z_2/Z_1) + \cos \bar{\theta}_T / \cos \theta_I} & \text{when } \bar{\theta}_T \text{ is real.} \\ 1 & \text{when } \bar{\theta}_T \text{ is imaginary} \end{cases} \quad \text{Eq. 27.3}$$

- Hence, using Eq. 27.2, we get the following relation for T_P :

$$T_P = \frac{4(Z_2/Z_1) \cos \bar{\theta}_T / \cos \theta_I}{[(Z_2/Z_1) + \cos \bar{\theta}_T / \cos \theta_I]^2} \quad \text{if } \bar{\theta}_T \text{ is real.} \quad \text{Eq.27.4}$$

$$T_P = 0 \quad \text{if } \bar{\theta}_T \text{ is imaginary} \quad \text{Eq.24 .5}$$

Power Coefficients

- As discussed earlier, $\overline{\theta_T}$ is real either when $c_2 < c_1$ or when $\vartheta_i < \vartheta_c$. In such a case if $Z_2/Z_1 = \cos \overline{\theta_T} / \cos \theta_i$, then Eq. 27.11 predicts that T_p will be 1.0 and if we use this result in Eq. 27.3, $R_p=0$. hence, under such conditions, all power is transmitted while no reflection occur.
- The angle of incidence ϑ_i associated with the condition $Z_2/Z_1 = \cos \overline{\theta_T} / \cos \theta_i$ (when $c_2 < c_1$ or $\vartheta_i < \vartheta_c$) is known as angle of intromission. For such a value of incidence angle, there is no reflection and all energy gets completely transmitted.

Angle of Intromission

- The value of this angle can be computed by using Eqs. 24.3 in conjunction with the condition $Z_2/Z_1 = \cos \bar{\theta}_T / \cos \theta_I$. Thus we get:

$$\begin{aligned}\sin \theta_I|_{intromission} &= \left[\frac{(Z_2/Z_1)^2 - 1}{(Z_2/Z_1)^2 - (c_2/c_1)^2} \right]^{1/2} \\ &= \left[\frac{1 - (Z_2/Z_1)^2}{1 - (\rho_1/\rho_2)^2} \right]^{1/2}\end{aligned}\quad \text{Eq. 27.6}$$

Normal Specific Acoustic Impedance

- We have seen through transmission line theory that when an acoustic wave hits a fluid boundary, it gets reflected in part and transmitted in part.
- Further, pressure at the interface has to be continuous. Since complex amplitude of pressure on incident side is $P_I(x, \omega) + P_R(x, \omega)$, while that for transmitted wave is $P_T(x, \omega)$, the requirement of pressure continuity implies:

$$[P_I(0, \omega) + P_R(0, \omega)]e^{j\omega t} = P_T(0, \omega)e^{j\omega t}$$

$$P_I + P_R = P_T$$

Eq. 27.6

Normal Specific Acoustic Impedance

- Also, at the interface (when $x=0$), normal velocity on either sides is equal. Thus:

$$U_I \cos \theta_I + U_R \cos \theta_R = U_T$$

- Since $\vartheta_I = \vartheta_R$ and $\overline{U_I} + \overline{U_R} = \overline{U}$ (net velocity in medium 1). Thus:

$$\overline{U} \cos \theta_I = U_T \cos \theta_T$$

Eq. 27.8

- Dividing Eq. 27.6 by Eq. 27.8, we get:

$$\frac{P_I + P_R}{\overline{U} \cos \theta_I} = \frac{P_T}{U_T \cos \theta_T} = \frac{Z_2}{\cos \theta_T}$$

Eq. 27.9

Normal Specific Acoustic Impedance

- Since the numerator of LHS of Eq. 27.9 is total pressure in media 1 at $x=0$ and $\bar{U} \cos \theta_i$ is normal velocity at $x=0$, the LHS is termed as “normal specific acoustic impedance”, Z_n . Thus:

$$\frac{P}{\bar{U} \cos \theta_i} = Z_n = \frac{P_T}{U_T \cos \theta_T} \quad \text{Eq. 27.10}$$

- Further, we note that $P_R/P_I=R$ and $U_R/U_I=-R$ from transmission line equation. Thus:

$$Z_n = \frac{P}{\bar{U} \cos \theta_i} = \frac{P_I + P_R}{(U_I + U_R) \cos \theta_i} = \frac{(1 + R)Z_\phi}{(1 - R) \cos \theta_i}$$

Normal Specific Acoustic Impedance

- Solving for R from above we get:

$$R = \frac{Z_n - Z_1 / \cos \theta_I}{Z_n + Z_1 / \cos \theta_I} \quad \text{Eq. 27.11}$$

- For special case $\theta_I = 0^\circ$ (normal incidence) and $Z_n = Z_2$, Eq. 27.12 reduces to:

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad \text{Eq. 27.13}$$

Normal Specific Acoustic Impedance

- Here we note that Eq.27.13 is same as Eq. 23.7.
- Finally it should be noted that Z_n is a complex entity and may have real as well as imaginary components. Hence we can express Z_n as:

$$Z_n = r_n + jx_n \quad \text{Eq. 27.14}$$

where,

r_n = normal specific acoustic resistance.

x_n = normal specific acoustic reactance.

References

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- Introduction to Acoustics, Finch Robert D., Pearson Prentice Hall, 2005.
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- Sound and Structural Vibration, Fahy Frank, et al, 2nd ed., Academic Press 2007.