

Sound Propagation through Media

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LECTURE-26

TRANSMISSION BETWEEN TWO FLUIDS AT OBLIQUE INCIDENCE

Introduction

- In this lecture, we continue to explore propagation of sound waves through two fluid media at oblique incidence angles.
- Till so far, we have developed expressions for propagation vectors, and also for transmission and reflection coefficients.
- Next, we explore how an obliquely incident wave travels through two media, using the results obtained in previous lectures.

Observations

- If $c_2 < c_1$ then the term under $\sqrt{}$ sign in Eq. 25.11 is always positive. This implies that $\cos \overline{\theta_T}$ is always real. It also implies that $\vartheta_T < \vartheta_i$. Thus, the transmitted beam bends towards normal for all angles of incidence, as long as $c_2 < c_1$.
- If $c_2 = c_1$, then $\overline{\theta_T}$ is real and $\vartheta_T = \vartheta_i$. Thus there is no change in direction of transmitted wave, because $c_2 = c_1$.
- If $c_2 > c_1$, then the term under $\sqrt{}$ sign in Eq. 25.11 can either be positive or negative.

Observations

- For v term to be positive (when $c_2/c_1 > 0$), we see that the value of incidence angle ϑ_i should not exceed $\sin^{-1}(c_1/c_2)$. This value of incidence angle is known as critical angle, i.e. ϑ_c . Thus; if $\vartheta_i < \vartheta_c$, where:

$$\sin \theta_c = \frac{c_1}{c_2}$$

thus v is always positive.

- In such a case, $\overline{\theta_T}$ is once again real. However, ϑ_T is greater than angle of incidence, i.e. the transmitted wave tends “away” from the normal for all angles of incidence.

Observations

- Finally, we explore the case when v term in Eq. 25.11 is negative. This occurs when $c_1 < c_2$ and $\vartheta_i > \vartheta_c$. In such a case;

$$\cos \overline{\theta}_T = \sqrt{1 - (c_2/c_1)^2 \sin^2 \theta_I} = \text{Imaginary numbers}$$

$$\cos \overline{\theta}_T = \pm j[(c_2/c_1)^2 \sin^2 \theta_I - 1]^{1/2} \quad \text{Eq.26.1}$$

- Also,

$$\sin \overline{\theta}_T = (c_2/c_1) \sin \theta_I \quad (\text{from Eq. 26.11}) \quad \text{Eq. 26.2}$$

here $\sin \overline{\theta}_T$ is real, but its value exceeds unity.

Observations

- Substituting Eqs. 26.1 and 26.2 in Eq. 24.5, we get:

$$\begin{aligned} P_t(x, y, t) &= P_T e^{j\omega t} \left[e^{\pm j^2 \frac{2\pi}{\lambda_1} x \sqrt{(c_2/c_1)^2 \sin^2 \theta_I - 1}} \cdot e^{-j \frac{2\pi}{\lambda_2} y \left(\frac{c_2}{c_1}\right) \sin \theta_I} \right] \\ &= P_T e^{j\omega t} \left[e^{\pm \frac{2\pi}{\lambda_2} x \sqrt{(c_2/c_1)^2 \sin^2 \theta_I - 1}} \cdot e^{-j \frac{2\pi}{\lambda_1} y \sin \theta_I} \right] \end{aligned} \quad \text{Eq. 26.3}$$

- Equation 26.3 can be rewritten as:

$$P_t(x, y, t) = P_T e^{j\omega t} \left[e^{\pm r x} \cdot e^{-\frac{2\pi}{\lambda_1} y \sin \theta_I} \right] \quad \text{Eq. 26.4}$$

where,

$$r = \frac{2\pi}{\lambda_2} \sqrt{(c_2/c_1)^2 \sin^2 \theta_I - 1}$$

Observations

- In Eq. 26.3, if the sign before r is +ve, then it implies that the strength of the wave increases with x in exponential fashion. This is not physically possible. Hence the solution is:

$$P_t(x, y, t) = P_T e^{j\omega t} \left[e^{-rx} e^{-\frac{2\pi y}{\lambda_1} \sin \theta_I} \right] \quad \text{Eq. 26.4}$$

- Equation 26.4 shows that if $\vartheta_i > \vartheta_c$ and $c_2 > c_1$, then:
 - For fixed value of x , the transmitted wave progresses harmonically in y direction, and its wavelength is λ_1 .
 - The transmitted wave does not progress harmonically in x -direction. Rather in x -direction, its strength decays exponentially.

Observations

- As ϑ_T is imaginary, the value of R as per Eq. 25.10 is complex. Further, In such case, the denominator and numerator of R are complex conjugates of each other implying that their magnitude are same and $|R| = 1$. Thus,

$$R = e^{j\phi}$$

Eq. 26.6

where,

$$\phi = 2 \tan^{-1} \left[(\rho_1 / \rho_1) \sqrt{(\cos \theta_c / \cos \theta_I)^2 - 1} \right]$$

Eq. 26.7

- Thus, for $\vartheta_i > \vartheta_c$ and $c_2 > c_1$, the reflected wave same complex amplitude as that of incident wave.

Observations

- Further, in such a case, the transmitted wave exists but it propagates only in y-direction. This implies that there is no energy being propagated into sound medium and transmitted wave “sticks” to the interface of two media.
- Still further, if ϑ_i is slightly more than ϑ_c , then φ (phase of reflected wave) is approximately zero and R equals approximately +1.0. Thus, in such a case, the interface acts like a rigid boundary.
- As ϑ_i increases with respect to ϑ_c , $(\cos \theta_c / \cos \theta_I)^2$ exceeds unity, and in extreme case, φ approaches π . In such a case, the reflection coefficient approaches -1 and interface condition is similar to that present at the end of an open tube.

Power Coefficients

- Next we calculate power reflection coefficient (R_p) and power transmission coefficient (T_p).
- Since, the cross-sectional area of incident and reflected wave is same, we can state:

$$R_p = R_I = |R|^2 \quad \text{Eq. 26.8}$$

- Also, from principle of conservation of energy, we can state:

$$T_p = 1 - R_p = 1 - |R|^2 \quad \text{Eq. 26.9}$$

Power Coefficients

- Using Eqs. 26.18 and 26.9, we can write:

$$R_P = \begin{cases} \frac{(Z_2/Z_1) - \cos \bar{\theta}_T / \cos \theta_I}{(Z_2/Z_1) + \cos \bar{\theta}_T / \cos \theta_I} & \text{when } \bar{\theta}_T \text{ is real.} \\ 1 & \text{when } \bar{\theta}_T \text{ is imaginary} \end{cases} \quad \text{Eq. 26.10}$$

- Hence, using Eq. 26.9, we get the following relation for T_P :

$$T_P = \frac{4(Z_2/Z_1) \cos \bar{\theta}_T / \cos \theta_I}{[(Z_2/Z_1) + \cos \bar{\theta}_T / \cos \theta_I]^2} \quad \text{if } \bar{\theta}_T \text{ is real.} \quad \text{Eq.26.11}$$

$$T_P = 0 \quad \text{if } \bar{\theta}_T \text{ is imaginary} \quad \text{Eq.24 .12}$$

Power Coefficients

- As discussed earlier, $\overline{\theta_T}$ is real either when $c_2 < c_1$ or when $\vartheta_i < \vartheta_c$. In such a case if $Z_2/Z_1 = \cos \overline{\theta_T} / \cos \theta_i$, then Eq. 26.11 predicts that T_p will be 1.0 and if we use this result in Eq. 26.9, $R_p=0$. hence, under such conditions, all power is transmitted while no reflection occur.
- The angle of incidence ϑ_i associated with the condition $Z_2/Z_1 = \cos \overline{\theta_T} / \cos \theta_i$ (when $c_2 < c_1$ or $\vartheta_i < \vartheta_c$) is known as angle of intromission. For such a value of incidence angle, there is no reflection and all energy gets completely transmitted.

Angle of Intromission

- The value of this angle can be computed by using Eqs. 24.3 in conjunction with the condition $Z_2/Z_1 = \cos \bar{\theta}_T / \cos \theta_I$. Thus we get:

$$\begin{aligned}\sin \theta_I|_{\text{intromission}} &= \left[\frac{(Z_2/Z_1)^2 - 1}{(Z_2/Z_1)^2 - (c_2/c_1)^2} \right]^{1/2} \\ &= \left[\frac{1 - (Z_2/Z_1)^2}{1 - (\rho_1/\rho_2)^2} \right]^{1/2}\end{aligned}\quad \text{Eq. 26.13}$$

References

- Acoustics, Beranek Leo L., Acoustical Society of America, 1993.
- Introduction to Acoustics, Finch Robert D., Pearson Prentice Hall, 2005.
- Fundamentals of Acoustics, Kinsler Lawrence E., et al, 4th ed., John Wiley & Sons, 2005.
- Sound and Structural Vibration, Fahy Frank, et al, 2nd ed., Academic Press 2007.