Course Name Convective Heat and Mass Transfer

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Module 1:Preliminary Concepts and BasicEquations

Lecture 1: Reynolds Transport Theorem and Incompressible Flow

The Lecture Contains:

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Reynolds Transport Theorem :

The rate of change of N for a system equals the sum of change of N inside any arbitrary control volume in continuum and the rate of efflux of N across the control surface. This can be represented as

$$\frac{DN}{Dt} = \frac{\partial}{\partial t}\int\int\int_{\forall}\eta(\rho d\forall) + \int\int_{A_o}\eta(\rho \mathbf{V}.d\mathbf{A}) \quad \textbf{(1.1)}$$

N= Extensive Property; η = specific property

Mass conservation **(Continuity Equation)**

$$\frac{DM}{Dt} = \frac{\partial}{\partial t}\int\int\int_{\forall}\rho d\forall + \int\int_{A_o}\rho \mathbf{V}.d\mathbf{A} \quad \textbf{(1.2)}$$

$$\int\int\int_{\forall}\frac{\partial \rho}{\partial t}d\forall + \int\int\int_{\forall}\{\nabla.(\rho\mathbf{V})\}d\forall \quad \textbf{(1.3)}$$

$$\int\int\int_{\forall}\left[\frac{\partial \rho}{\partial t} + \nabla.(\rho\mathbf{V})\right]d\forall = 0 \quad \textbf{(1.4)}$$

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho\mathbf{V}) = 0 \quad \textbf{(1.5)}$$

$$\frac{\partial \rho}{\partial t} + (\mathbf{V}.\nabla)\rho + \rho(\nabla.\mathbf{V}) = 0$$

The Genral Form of the above equation is

$$\frac{D\rho}{Dt} + \rho(\nabla.\mathbf{V}) = 0 \quad \textbf{(1.6)}$$

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For incompress Flows, $(D\rho/Dt) = 0$

Governing Equation

$$\rho(\nabla.\mathbf{V}) = 0 \qquad \textbf{(1.7)}$$

So the contunity equation for incompressible **steady/unsteady flow**

$$\nabla.\mathbf{V} = 0 \qquad \textbf{(1.8)}$$

Compressible and Incompressible Flows :

Let us consider change of volume of a fluid under the action of external forces.

$$E = \frac{-\Delta p}{\Delta\forall/\forall} \qquad \textbf{(1.9)}$$

E for water is $2 \times 10^6\ kN/m^2$, E for air $= 101\ kN/m^2$. **Air is 20,000 times** more compressible than water.

The Mass community may be written as

$$(\forall + \Delta\forall)(\rho + \Delta\rho) = \forall\rho \qquad \textbf{(1.10)}$$

which gives

$$\frac{\Delta\forall}{\forall} = -\frac{\Delta\rho}{\rho} \qquad \textbf{(1.11)}$$

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Now, the modulus of **Elasticity, E is expressed as**

$$E \approx \frac{-\Delta p}{-(\Delta\rho/\rho)} \approx \frac{\Delta p}{\Delta\rho/\rho} \qquad \textbf{(1.12)}$$

$$\frac{\Delta\rho}{\rho} \approx \frac{\Delta p}{E} \approx \frac{1}{2}\frac{\rho V^2}{E} \quad [\,V \text{ is the velocity}] \qquad \textbf{(1.13)}$$

$$\frac{\Delta\rho}{\rho} \sim \frac{1}{2}\frac{V^2}{a^2} \qquad \textbf{(1.14)}$$

or

where, $a = \sqrt{\frac{E}{\rho}}$ = **Local acoustic speed**

or,

$$\frac{\Delta\rho}{\rho} \sim \frac{1}{2}M^2 \qquad \textbf{(1.15)}$$

where, **M = Mach number**

Considering a maximum relative change in density of 5 percent as the criterion of an incompressible flow, the upper limit of **Mach number is** $M \approx 0.334$;

which means, flow of air upto a velocity of 110 m/s under standard condition, can be considered as incompressible flow.