Introduction to Bode Plot

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This Lecture Contains

- Introduction to Bode Plot
- Bode Plot of a First Order System
- Bode plot of Higher Order System
- Gain and Phase Margin
- Assignment

Introduction to Bode Plot

- *So far we have designed compensators based on Root-Locus technique and time domain response of a system.*
- *We have used Frequency Response only to find the stability of a dynamic system – remember Nyquist Stability Criteria*
- *Nyquist plot presents the Real vs. Imaginary plot of the open loop transfer function, the nature of which has given us clues on stability of a system. However, we have not obtained a direct plot of Magnitude and Phase of a Control System with respect to frequency.*

- **Bode Plot deals with the frequency response of a system simultaneously in terms of magnitude and phase. More precisely, the log-magnitude and phase frequency response curves are known as Bode Plots. Such plots are useful due to the following reasons:**
  - *For designing lead compensators*
  - *For finding stability, gain and phase margin*
  - *For system identification from the frequency response*

Bode Plot based on Asymptotic Approximation

Consider a generalized transfer function as:

$$T(s)=\frac{K\prod_{i=1.}^{m}(s+z_i)}{s^k\prod_{j=1}^{n-k}(s+p_j)}$$

The magnitude (in terms of decibel) and phase of the transfer function are:

$$20\log|T(s)|=20\log K+\sum_{i=1}^{m}20\log|s+z_i|-20\log|s^k|-\sum_{j=1}^{n-k}20\log|s+p_j|$$

$$\angle T(s)_{\omega\to\infty}=(m-n)\frac{\pi}{2}$$

Bode Plot of *(s+p)*

- **Consider a transfer function *T(s)* = *s+p***
- **The frequency response may be obtained by applying *s=jω.***
- **Accordingly, we may write**

$$T(j\omega) = j\omega + p = p(1 + j\frac{\omega}{p})$$

- **Now, for very low value of the frequency:**

$$|T(j\omega)| \approx p$$

$$20\log|T(j\omega)| \approx 20\log(p)$$

- **Let us look at the system behavior for higher value of the frequency (say ω/p >1).**

Bode plot for (s+p) – contd.

- **For such cases –**

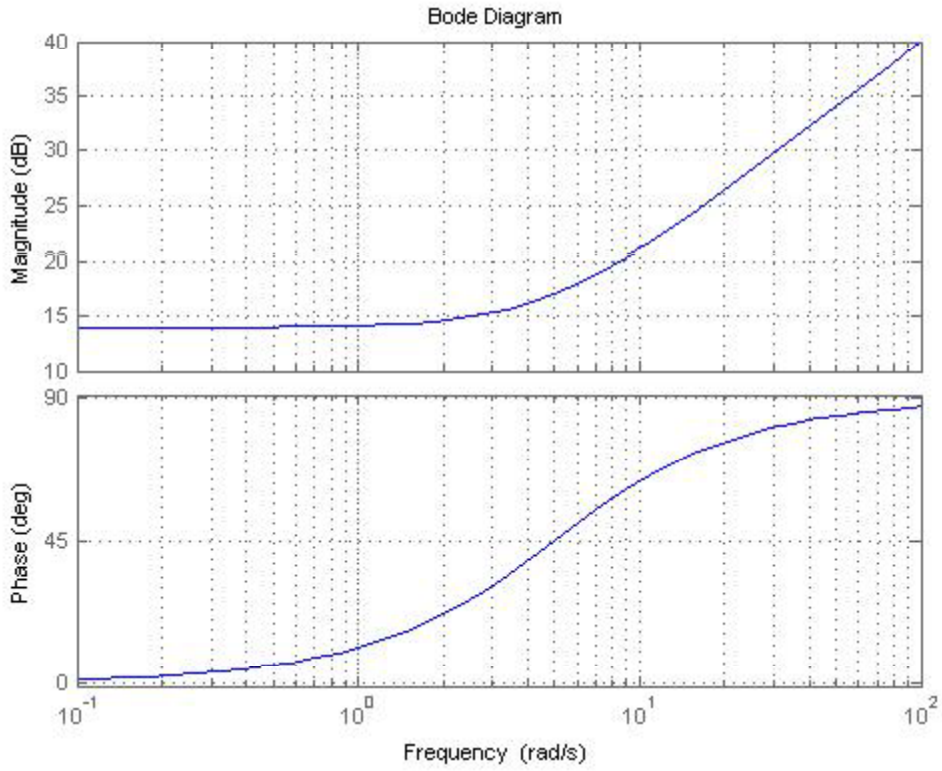
$$T(j\omega) \approx p(j\frac{\omega}{p})$$

$$20\log(T) \approx 20\log(p) + 20\log(\frac{\omega}{p}) \approx 20\log(\omega)$$

- **In other words, the magnitude of the transfer function could be considered to be constant (20 log(p)) till ω=p and then increasing at the rate of 20dB per decade.**
- **How about the phase. We know that the phase as ω tends to ∞ will be 90^0 . At lower frequencies we can use the relationship stated earlier and find the phase to be close to zero till ω is about 0.1a and then increase at the rate of 45^0 /decade till it reaches 90^0 .**

- **The actual behavior is plotted in the next slide.**

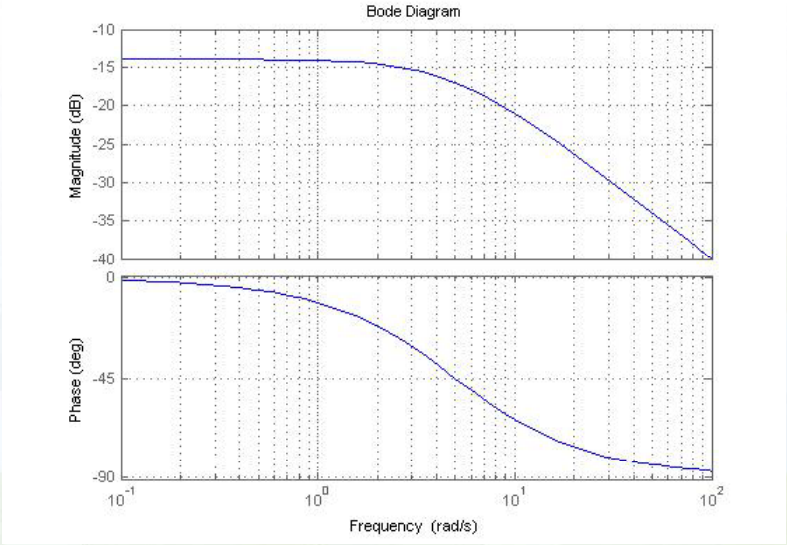
Bode Plot for (s+5)



Bode Plot for [1/(s+p)]

- **In this case, one can follow a similar procedure to find the asymptotic behavior. It can be shown that for low frequency the magnitude is close to 20 log(1/p) and beyond p, it decreases at the rate of 20dB per decade. The phase plot will show that the initial phase to be close to zero and then decrease at the rate of 45 degree per decade until reaches -90^0 as the frequency goes beyond 10p**

- **The actual plot for T(s) = 1/(s+5) is shown below:**



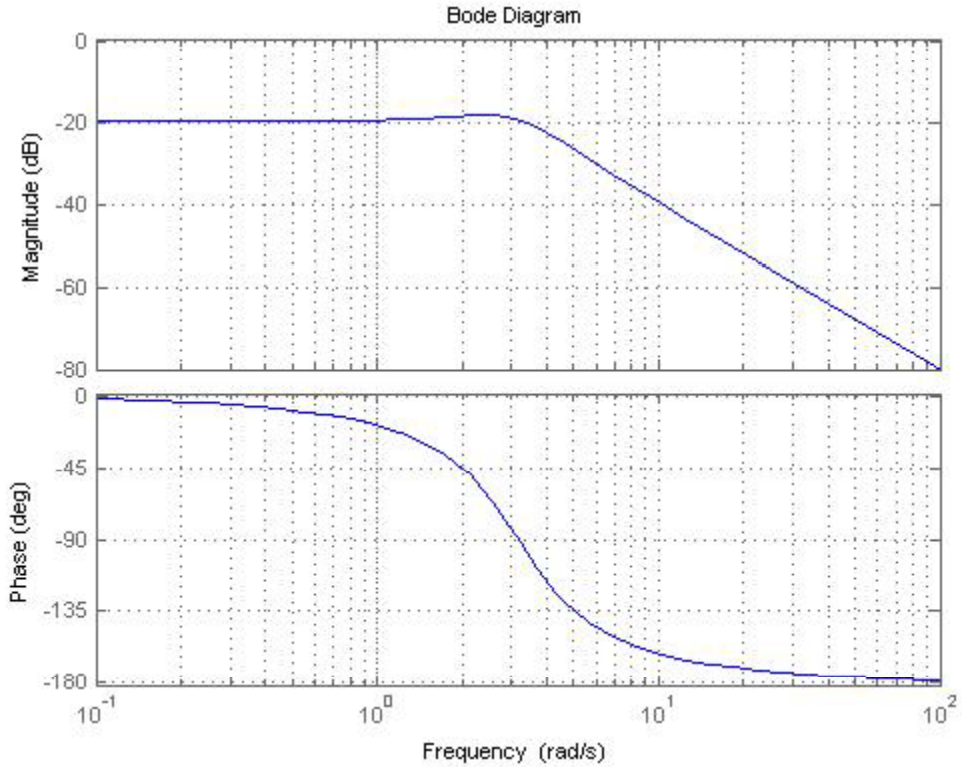
A few more comments on Bode Plot

- For a cascaded system having two poles, similar arguments could be placed and the change of slope corresponding to each of the poles at the rate of -20dB per decade for the magnitude plot and -45^0 per decade for the phase plot could be applied.

- For a second order system, again asymptotic plot will show the magnitude to be reducing at the rate of -40dB per decade and the phase plot will show -90^0 per decade decay from the initial value till it reaches -180^0 .

- A typical Bode-plot of a second order system is shown in the next slide. You can use MATLAB for Bode plot by first defining the' transfer function' and then using the command 'bode('transfer function')'.

Bode plot for [1/(s^2 + 3s + 10)]



Gain and Phase Margin using Bode Plot

- From Nyquist Criteria you know that instability occurs if there is encirclement of -1 corresponding to a phase of 180^0 .
- This implies that the for stability the magnitude of the transfer function must be less than unity at a frequency which corresponds to 180^0 phase. In fact the actual Gain in dB corresponding to this frequency provides the gain margin.
- In a similar manner the phase value corresponding to the frequency where gain of the system is 0 dB provides the phase margin.
- You can use the 'margin' command in MATLAB to obtain both the Bode plot, Gain and Phase Margin.

Assignment

- **Consider a unity feedback system with a plant transfer function:**

$$G(s)=\frac{(s+6)}{(s+2)(s+4)(s+7)(s+8)}$$

- **Input the transfer function in MATLAB and sketch the bode plot.**
- **Find out the Gain and Phase Margin and comment on the stability of the system.**

Special References for this lecture

- **Feedback Control of Dynamic Systems, Frankline, Powell and Emami, Pearson**
- ***Control Systems Engineering* – Norman S Nise, John Wiley & Sons**
- ***Design of Feedback Control Systems* – Stefani, Shahian, Savant, Hostetter**

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