

9.1 An aircraft instrument weighing 100 N is to be isolated from engines vibrations ranging from 25Hz-40Hz . What static deflection must the isolators have for achieving 85% isolation.

mass of instrument $m=100\text{N}$

frequency of isolation= 25-40 Hz

isolation =85%

hence transmissibility= $1-0.85=0.15$

$$Tr = \frac{\sqrt{1 + \left(2\xi \frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi \frac{\omega}{\omega_n}\right)^2}}$$

If we assume $\xi = 0$ (very low damping)

$$Tr = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}$$

$$0.15 = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}$$

solving

$$\frac{\omega}{\omega_n} \geq 2.768$$

If the transmissibility at lower frequency is less , higher frequencies also even lesser force transmission will be there.

Now let $\omega = 25\text{Hz}$

$$\therefore \omega_n = \frac{(2\pi)(25)}{2.768} = 56.75 \text{ rad/s}$$

For a spring-mass model

let δ_{st} be the static

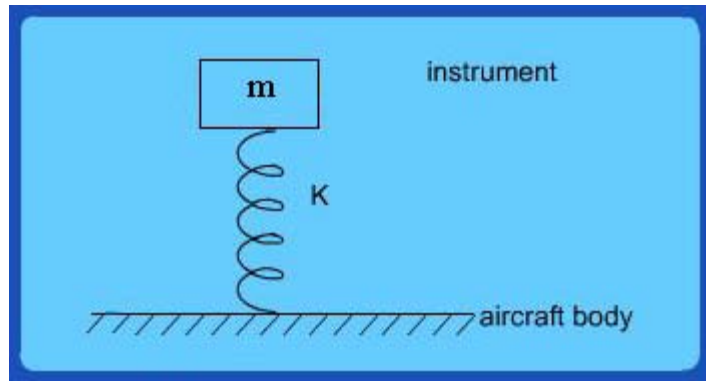
deflection of the spring under the weight of instrument.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\left(\frac{l}{m}\right)\left(\frac{mg}{\delta_{st}}\right)} = \sqrt{\frac{g}{\delta_{st}}}$$

$$\therefore \sqrt{\frac{g}{\delta_{st}}} = 56.75$$

$$\therefore \delta_{st} = \frac{9.8}{(56.75)^2} * 1000 \text{ mm}$$

$$= 3.043 \text{ mm}$$

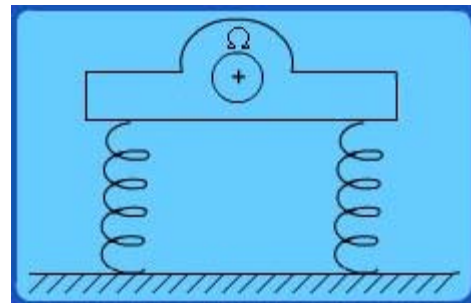


9.2 An industrial machine of 450kg is supported on springs with a static deflection of 0.50cm. If the machine has rotating unbalance of 0.2kgm. Determine a) the force transmitted to floor at 1200rpm and b) the dynamic amplitude at this speed.

Given : mass of machine $m=450\text{kg}$

$$\Delta=0.5\text{mm}$$

$$K = \frac{mg}{\Delta} = \frac{450 * 9.81}{0.005} = 882900 \text{ N/m}$$



$$\Omega = 1200\text{RPM} = \left(\frac{1200}{60}\right)(2)(\pi) = 125.66 \text{ rad / s}$$

In absence of damping

$$Tr = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{882900 / 450} = 44.29 \text{ rad / s}$$

$$Tr=0.14$$

$$F = m\omega^2 e = (0.2\text{kg} \cdot \text{m})(125.66)^2$$

$$= 3158 \text{ N}$$

$$F_{tr} = (0.14)(3158) = 442 \text{ N}$$

9.3 The rotor of a turbine 15 kg in mass, is supported at the midspan of a shaft with bearings 0.5m apart. The rotor is known to have an unbalance of 0.2kgm . Determine the forces exerted on the bearings at a speed of 500 rpm if the diameter of the steel shaft is 20 mm.

For a simply supported shaft, at its centre the stiffness is

$$k = \frac{48EI}{\ell^3}$$

$$I = \frac{\pi}{64}d^4 = \frac{\pi}{64} \times (0.02)^4$$

$$k = \frac{48 \times 2.1 \times 10^{11} \times \frac{\pi}{64} \times (0.02)^4}{(0.5)^3} = 633345 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{633345}{15}} = 205.5 \text{ rad/s} = 1962 \text{ RPM}$$

$$\frac{\omega}{\omega_n} = \frac{500}{1962} = 0.255$$

$$\frac{y}{\ell} = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi \frac{\omega}{\omega_n}\right)^2}}$$

Assuming negligible damping, $\frac{x}{e} = \frac{(0.255)^2}{1 - (0.255)^2} = 0.069$

Now force due to whirling is centrifugal force due to rotating imbalance.

$$F_{CFI} = m(e + r) \omega^2$$

$$e+r = 1.069 e$$

$$F_{\text{cent}} = m\omega \times 1.069 \times \omega^2$$

$$= 1.069 \times 0.2 \times \left(\frac{500 \times 2 \times \pi}{60} \right)^2 = 586 \text{ N}$$

therefore force on bearings = $586 / 2 = 293 \text{ N}$