ME-662 CONVECTIVE HEAT AND MASS TRANSFER

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LECTURE-9 SIMILARITY SOLN TO TEMP BL - II

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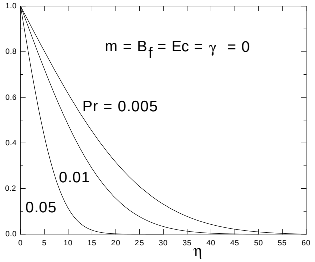
$$\theta'' + Pr \left[(\frac{m+1}{2})\, f\, \theta' - \gamma\, f'\, \theta + 2\, Ec\, (f'')^2 \right] = 0 \qquad (1)$$

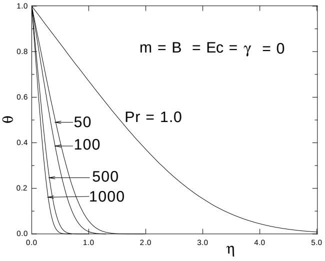
$$\theta(0) = 1 \text{ and } \theta(\infty) = 0$$

Similarity Solutions to study effect of

1. High and Low Prandtl Numbers (Pr)
2. Pressure Gradient (m)
3. Wall Temperature Variation (γ)
4. Viscous Dissipation (Ec)

Very Low and High Pr - L9($\frac{1}{12}$)





Liquid Metals

Pr	Δ^*	$-\theta'(0)$
0.005	53.3	0.04
0.01	39.3	0.0564
0.05	17.7	0.126

$Nu_x = 0.564\,(Re_x\ Pr)^{0.5}$

Oils

Pr	Δ^*	$-\theta'(0)$
100	1.066	1.57
500	0.624	2.68
1000	0.495	3.28

$Nu_x = 0.339\ Re_x^{0.5}\ Pr^{0.33}$ (see later)

Effect of m - ($B_f = \gamma = Ec = 0$) - L9($\frac{2}{12}$)

$$\theta^{''} + Pr\,(\frac{m+1}{2})\,f\,\theta^{'} = 0 \quad \text{or} \quad \frac{d\theta^{'}}{\theta^{'}} = -Pr\,(\frac{m+1}{2})\,f \qquad (2)$$

Integration gives

$$\begin{aligned}
\Big[\ln\,(\theta^{'})\Big]_0^{\eta} &= -Pr\,(\frac{m+1}{2})\,\int_0^{\eta} f\,d\eta \\
\theta^{'} &= \theta^{'}(0)\,\exp\left[-Pr(\frac{m+1}{2})\,\int_0^{\eta} f\,d\eta\right] \\
\theta &= \theta^{'}(0)\,\int_0^{\eta} \exp\left[-Pr(\frac{m+1}{2})\,\int_0^{\eta} f\,d\eta\right] d\eta + C_1
\end{aligned}$$

But, $\theta(0) = 1$. Hence, $C_1 = 1$. Also, $\theta(\infty) = 0$. Hence,

$$\theta^{'}(0) = -\,\left\{\int_0^{\infty} \exp\left[-Pr(\frac{m+1}{2})\,\int_0^{\eta} f\,d\eta\right] d\eta\right\}^{-1} \qquad (3)$$

Effect of m - Moderate Pr) - L9($\frac{3}{12}$)

$(B_f = \gamma = Ec = 0)$ - H T Coef $h_x \propto x^{(\frac{m-1}{2})}$. For stagnation point, h_x = Const.

m	-0.085	-0.065	-0.04	0.0	0.33	1.0	4.0
				Pr=0.7			
$-\theta'(0)$	0.22	0.25	0.27	0.29	0.38	0.49	0.81
Δ^*	6.76	6.21	5.93	5.60	5.24	3.55	2.19
				Pr=5			
$-\theta'(0)$	0.40	0.47	0.52	0.57	0.79	1.03	1.71
Δ^*	3.67	3.21	2.98	2.73	2.07	1.59	0.98
				Pr=10			
$-\theta'(0)$	0.49	0.59	0.65	0.72	1.00	1.32	2.18
Δ^*	2.99	2.58	2.37	2.15	1.59	1.23	0.77
				Pr=25			
$-\theta'(0)$	0.64	0.79	0.88	0.98	1.37	1.81	3.10
Δ^*	2.31	1.95	1.76	1.59	1.19	0.92	0.56

Correlations: Effect of m - $Pr >> 1$ L9($\frac{4}{12}$)

For $Pr >> 1$, $\Delta << \delta$. Hence, $f^{'}(\eta) \simeq f^{''}(0, m)\, \eta$. And, $f(\eta) \simeq f^{''}(0, m)\, \eta^2/2$. Then

$$\begin{aligned}\theta^{'}(0) &= -\left\{\int_0^{\infty} \exp\left[-Pr(\frac{m+1}{2})\int_0^{\eta} f^{''}(0, m)\,\frac{\eta^2}{2}\, d\eta\right] d\eta\right\}^{-1} \\ &= -\left[\int_0^{\infty} \exp\left\{-(\frac{Pr\,(m+1) f^{''}(0, m)}{12})\,\eta^3\right\} d\eta\right]^{-1}\end{aligned}$$

Using definition $\Gamma(n) = \int_0^{\infty} x^{n-1}\, e^{-x} dx$, it can be shown that

$$Nu_x\, Re_x^{-0.5} = -\theta^{'}(0) = \left\{\frac{Pr\,(m+1) f^{''}(0, m)}{12}\right\}^{0.33} \times \frac{1}{\Gamma(4/3)}$$

where $\Gamma(4/3) \simeq 0.893$. Hence, for m = 0, with $f^{''}(0) = 0.33$, $Nu_x \simeq 0.339\, Re_x^{0.5}\, Pr^{0.33}$

Correlations: Effect of m - $Pr << 1$ L9($\frac{5}{12}$)

For $Pr << 1$, $\Delta >> \delta$. Hence, $f^{'}(\eta) \simeq 1.0$. And, $f(\eta) \simeq \eta$. Then

$$\theta^{'}(0) = -\left\{\int_0^\infty \exp\left[-Pr(\frac{m+1}{2}) \int_0^\eta \eta \, d\eta\right] d\eta\right\}^{-1}$$
$$= -\left[\int_0^\infty \exp\left\{-(\frac{Pr\,(m+1)}{4})\,\eta^2\right\} d\eta\right]^{-1}$$

Using definition erf(x) = $\frac{2}{\sqrt{\pi}} \int_0^x e^{-\eta^2} d\eta$ and erf(∞) = 1.0

$$Nu_x \, Re_x^{-0.5} = -\theta^{'}(0) = \sqrt{\frac{Pr(m+1)}{\pi}}$$

Hence, for m = 0, $Nu_x = 0.564 \, (Re_x \, Pr)^{0.5}$

Effect of $T_w - T_\infty = C\, x^\gamma$ - L9($\frac{6}{12}$)

$m = B_f = Ec = 0$, $\theta'' + Pr\, \left[0.5\, f\, \theta' - \gamma\, f'\, \theta\right] = 0$

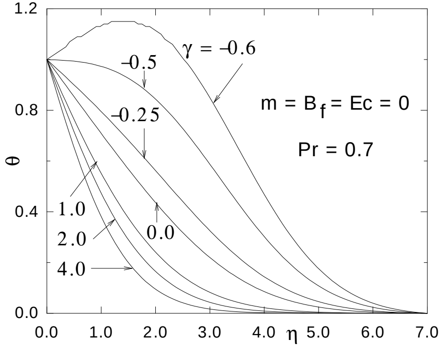
γ	Pr	0.7	5.0	10.0	25.0
4.0	$-\theta'(0)$	0.72	1.38	1.74	2.36
2.0	$-\theta'(0)$	0.582	1.12	1.41	1.91
1.0	$-\theta'(0)$	0.478	0.925	1.16	1.58
0.3	$-\theta'(0)$	0.366	0.713	0.898	1.22
0.0	$-\theta'(0)$	0.2913	0.572	0.721	0.976
-0.25	$-\theta'(0)$	0.195	0.388	0.489	0.662
-0.5	$-\theta'(0)$	0.0	0.0	0.0	0.0
-0.6	$-\theta'(0)$	-0.16	-0.45	-0.59	-0.84

Since Ec = 0, γ can take arbitrary values.

Special Case $\gamma < 0$ - L9($\frac{7}{12}$)

1. For $\gamma = -0.5$, temperature gradient at the wall is zero (adiabatic case) although $T_w > T_\infty$.
2. Since negative value of γ implies decreasing T_w with increasing x, the fluid particles close to the surface at a particular x arrive from upstream region of higher surface temperature.
3. These hotter fluid particles may thus inhibit any heat transfer from the surface to the cooler free stream.
4. By the same reasoning, for $\gamma < -0.5$, $\theta > 1$ at some distance close to the surface and, heat will flow **into** the surface even if $T_w > T_\infty$.
   Hence, H T Coef < 0 (or, negative).

Effect of γ - Profiles - L9($\frac{8}{12}$)



1. Notice that at $\gamma = -0.6$, θ exceeds wall value $\theta(0) = 1$
2. At $\gamma = -0.5$, $\theta'(0) = 0$

Viscous Dissipation - L9($\frac{9}{12}$)

$m = B_f = \gamma = 0$ and Pr = 0.7

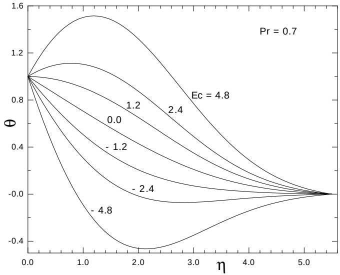
$\theta'' + 0.7\ \left[0.5\ f\ \theta' + 2\ Ec\ (f'')^2\right] = 0$

$Ec = (U_\infty^2/2)/(Cp(T_w - T_\infty))$ $\quad \theta = (T - T_\infty)/(T_w - T_\infty)$

Ec	-4.8	-2.4	-1.2	0.0	1.2	2.4	4.8
$-\theta'(0)$	1.458	0.875	0.583	0.292	0.004	-0.291	-0.874
Δ^*	0.861	2.70	4.84	5.26	5.35	5.39	5.43

1. Ec < 0 implies that $T_w < T_\infty$
2. Ec > 0 implies that $T_w > T_\infty$
3. For Ec = 1.2, $-\theta'(0) \simeq 0$ (adiabatic case)

Effect of Ec - Profiles - L9($\frac{9}{12}$)



1. For Ec < -1.2, $\theta < 0$ indicating that $T(\eta) > T_\infty$ *within* BL
2. For *Ec* > 1.2, $-\theta'(0) < 0$. Hence, $h_x < 0$ even when $T_w > T_\infty$
3. Both are effects of *local* viscous heating due to $\mu\,(\partial u/\partial y)^2$

Effect of B_f - L9($\frac{10}{12}$)

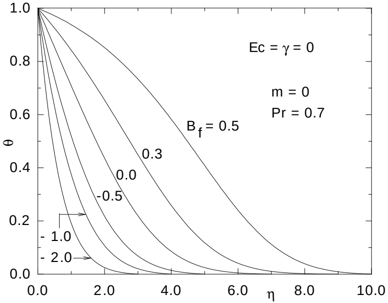
γ = Ec = 0, $\theta'' + 0.5\, Pr\,(m+1)\, f\,(m, B_f)\, \theta'$ =0

	Pr	0.5	0.7	1.0	0.5	0.7	1.0
B_f			m=0			m=1	
-2.0	$-\theta'(0)$	1.12	1.52	2.10	1.22	1.62	2.20
-1.0	$-\theta'(0)$	0.672	0.872	1.17	0.799	1.012	1.32
-0.5	$-\theta'(0)$	0.459	0.570	0.726	0.606	0.738	0.917
0.0	$-\theta'(0)$	0.259	0.2913	0.330	0.434	0.493	0.664
0.3	$-\theta'(0)$	0.142	0.141	0.134	0.338	0.366	0.392
0.5	$-\theta'(0)$	0.064	0.051	0.035	0.281	0.292	0.293
1.0	$-\theta'(0)$	NA	NA	NA	0.163	0.145	0.116

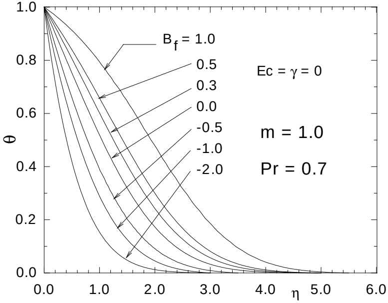
1. Only $0.5 < Pr < 1.0$ of interest (Gases)
2. Suction (negative B_f), increases H T by thinning BL
3. Blowing (positive B_f), decreases H T by thickening BL
4. For m = 0 and $B_f \geq 0.612$, Separation occurs

Effect of B_f - Profiles - L9($\frac{11}{12}$)

Flat Plate



Stagnation Point



Results for Pr = 0.5 and 1.0 are similar

Summary of Similarity Solns - L9($\frac{12}{12}$)

Velocity

1. Similarity Equations
   $f''' + (\frac{m+1}{2}) f f'' + m(1 - f'^2) = 0$
2. Boundary Conditions
   $f(0) = -B_f \, (\frac{2}{m+1})$
   $f'(0) = 0, f'(\infty) = 1$
3. Similarity Conditions
   $U_\infty = C \, x^m$
   $B_f = (\frac{V_w(x)}{U_\infty(x)}) \, Re_x^{0.5}$ = Const
   $\eta = y \times \sqrt{\frac{U_\infty}{\nu \, x}}$
4. Similarity Solutions
   $C_{f,x}(m, B_f) = 2 \, f''(0) \, Re_x^{-0.5}$

Temperature

1. Similarity Equations
   $\theta'' + Pr(\frac{m+1}{2}) \, f \, \theta'$ -
   $Pr \left[\gamma \, f' \, \theta - 2 \, Ec \, (f'')^2\right] = 0$
2. Boundary Conditions
   $\theta(0) = 1, \theta(\infty) = 0$
3. Similarity Conditions
   $T_w(x) - T_\infty = \Delta T_{ref} \, x^\gamma$
   $Ec_x = \frac{U_\infty^2(x)/2}{Cp(T_w(x) - T_\infty)}$ = Const
   If $Ec \neq 0$, $\gamma = 2m$
4. Similarity Solutions
   $Nu_x \, Re_x^{-0.5}(m, B_f, Pr, \gamma, Ec)$
   $= -\theta'(0)$