ME-662 CONVECTIVE HEAT AND MASS TRANSFER

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LECTURE-39 CONV M T - REYNOLDS FLOW MODEL - 1

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1. Wet Bulb Thermometer
2. Measurement of RH - Effect of Radiation
3. Evaporation from a Porous Surface
4. Evaporation from a Lake
5. Humidification - Internal Flow

All problems require use of Psychrometry and/or steam tables

Wet Bulb Thermometer - L39($\frac{1}{16}$)

Prob: A wet bulb thermometer records 15^0C when the dry bulb temperature is 27^0C. Calculate (a) RH of air and (b) compare with Carrier's correlation . Assume Le = 1 and take $c_{p,v}$ = 1.88 kJ/kg-K and $c_{p,a}$ = 1.005

Soln: Here, T_w = 15 and $T_\infty = T_{db}$ = 27. Since Le = 1,

$$B_m = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - \omega_{v,T}} = B_h = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{TL} + q_l/N_w} \quad \rightarrow \quad q_l = 0$$

Taking $T_{ref} = 0^0$C, λ_{ref} = 2503 kJ/kg, and from Steam Tables, $\omega_{v,w}$ = 0.01068, we have

$$\begin{aligned} h_{m,\infty} &= 1.005 \times 27 + \{(1.88 - 1.005) \times 27 + 2503\}\ \omega_{v,\infty} \\ h_{m,w} &= 15.075 + \{0.875 \times 15 + 2503\}\ 0.01068 = 41.95 \\ h_{TL} &= c_{p,l} \times T_w = 4.187 \times 15 = 62.805 \end{aligned}$$

Soln (Contd) - L39($\frac{2}{16}$)

Therefore

$$\frac{\omega_{v,\infty} - 0.01068}{0.01068 - 1} = \frac{27.135 + 2526.2\,\omega_{v,\infty} - 41.95}{41.95 - 62.805}$$

Solving,$\omega_{v,\infty}$ = 0.005936 . Hence, $W_\infty = \omega/(1-\omega)$ = 0.00594 at 27^0C. From psychrometric chart, this value corresponds to RH = 27 % (Ans a) . Also, $B_m = B_h$ = 0.00479 (Very small).

Carrier's correlation is

$$p_{v,\infty} = p_{sat,w} - \frac{(p_{tot} - p_{sat,w})\,(T_{wb} - T_{db})}{1555 - T_{wb}} \;\rightarrow\; (T^0C)$$

where p_{tot} = 1 bar, W_w = 0.01068/(1 - 0.01068) = 0.0108 and $p_{sat,w} = W_w \times p_{tot}/0.622$ = 0.01736 bar.
Hence, substitution gives $p_{v,\infty}$ = 0.0097. Therefore

$$\omega_{v,\infty} = \frac{p_{v,\infty}}{1.61 \times p_{tot} - 0.61 \times p_{v,\infty}} = 0.00604 \quad \text{(Ans b)}$$

Measurement of RH - L39($\frac{3}{16}$)

Prob: Moist air flows through a duct whose walls are maintained at 50^0C. Dry and wet bulb thermometers placed in the duct record 70^0C and 25^0C respectively. Between thermometer bulb and air, h_{cof} = 17.5 W / m^2-K. Calculate RH of air (a) without radiation and (b) with radiation .

Soln: Part (a) Here, again W_w = 0.02, $\omega_{v,w}$ = 0.0196, $h_{m,w}$ = 74.61, h_{TL} = 105.7 and $h_{m,\infty}$ = 70.35 + 2564.25 $\omega_{v,\infty}$. Therfore, with q_l = 0, equating B_m and B_h,

$$\frac{\omega_{v,\infty} - 0.0196}{0.0196 - 1} = \frac{70.35 + 2564.25\,\omega_{v,\infty} - 74.61}{74.61 - 104.675}$$

Solving $\omega_{v,\infty}$ = 0.001446 (B = 0.0185). Therefore, $p_{v,\infty}$ = 0.002328 bar. But, from steam tables, $p_{v,sat}\,(T_\infty = 70^0\text{C})$ = 0.3119 bar.

Hence, RH = 0.002328 / 0.3119 = 0.746 % (Ans) .

Soln (Contd) - 1 - L39($\frac{4}{16}$)

Soln: Part (b) Here, $q_l = 0$ and $h_{TL} = h_T + q_{rad}/N_w$.

$$B_{mh} = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - \omega_{v,T}} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_T} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{TL} + q_{rad}/N_w}$$

Now, we determine true air temperature ($T_{a,true}$) allowing for radiation effects. Thus

$$\begin{aligned} h_{cof}\,(T_{a,true} - T_{db}) &= \sigma\,(T_{db}^4 - T_w^4) \\ 17.5\,(T_{a,true} - 343) &= 5.67 \times 10^{-8}\,(343^4 - 323^4) \quad \text{or} \\ T_{a,true} &= 352.6K = 79.6^0\text{C} \quad \text{and} \\ q_{rad} &= \sigma\,(T_w^4 - T_{wb}^4) \\ &= 5.67 \times 10^{-8}\,(323^4 - 298^4) = 170\ \frac{W}{m^2} \end{aligned}$$

Now, B_m and B_h are freshly evaluated (next slide)

Soln (Contd) - 2 - L39($\frac{5}{16}$)

Here, we take $T_{ref} = T_{wb}$ = 25 so that λ_{ref} = 2442.3 kJ/kg.

$$\begin{aligned} h_{m,\infty} &= c_{p,a}\,(T_{a,true} - T_{ref}) \\ &+ [(c_{p,v} - c_{p,a})\,(T_{a,true} - T_{ref}) + \lambda_{ref}]\;\omega_{v,\infty} \quad \text{kJ/kg} \\ &= 54.52 + 2490\,\omega_{v,\infty} \\ h_{m,w} &= \omega_{v,w}\,\lambda_{ref} = 0.0196 \times 2442.3 = 47.87 \quad \text{kJ/kg} \\ h_{TL} &= cp_l\,(T_{wb} - T_{ref}) = 0 \quad \text{kJ/kg} \end{aligned}$$

Also $\omega_{mean} \simeq 0.5\,(0.0196 + 0.001446) = 0.0105$.
Therefore, $c_{p,m}$ = 1.041 kJ/kg-K.
Hence, $g^* = h_{cof}/c_{pm}$ = 17.5 / 1041 = 0.01725 kg/m^2-s.
Now, since B is expected to be small,
we take $(q_{rad}/N_w) = q_{rad}/(g^*\,B)$, so that

$$B = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - \omega_{v,T}} = \frac{h_{m,\infty} - h_{m,w} - (q_{rad}/g^*)}{h_{m,w} - h_{TL}}$$

Soln (Contd) - 3 - L39($\frac{6}{16}$)

Substitutions give (q_{rad}/g^*) = 0.17 / 0.01725 = 9.885 kJ/kg. Hence

$$B = \frac{\omega_{v,\infty} - 0.0196}{0.0196 - 1} = \frac{54.52 + 2490\,\omega_{v,\infty} - 47.87 - 9.885}{47.85 - 0}$$

This gives $\omega_{v,\infty}$ = 0.001693 and B = 0.01826.
So, our assumption of small B is verified.
Thus, W_∞ = 0.001648 and $p_{v,\infty}$ = 0.00264 bar.
But, $p_{v,sat}$ $(T_\infty = 79.58^0$C) = 0.4739 bar.
Hence RH = 0.00264 / 0.4739 = 0.557 % (Ans) .

This shows that true RH is lower than that predicted by neglecting effect of radiation

Evaporation - High B - L39($\frac{7}{16}$)

Prob: Air at 1 bar, 27^0C and 90 % RH flows over a porous flat surface which is kept wet by supplying water. The plate temperature is maintained at 82.5^0C by supplying heat to it from the rear side. Calculate (a) Evaporation rate and (b) Heat flux to be supplied . Given: U_∞ = 3.0 m/s, Length L = 0.33 m and Width W = 1 m. Take k = 0.025 $\frac{W}{m-K}$, ρ_m = 1.37 $\frac{kg}{m^3}$, D = 0.162 $\frac{m^2}{hr}$, α = 0.153 $\frac{m^2}{hr}$ and ν = 0.103 $\frac{m^2}{hr}$

Soln: In this case,
$Re_L = 3 \times 0.33/(0.103/3600) = 34601.9 < 3 \times 10^5$.
Thus, we have a laminar BL. Also, Pr = 0.103 / 0.153 = 0.673 and Sc = 0.103 / 0.162 = 0.636.
Hence, St = 0.664 $\times Re_L^{-0.5} \times Pr^{-0.67}$ = 0.00465.
Therfore, $g^* = h_{cof}/cp_m = \rho_m\, U_\infty\, St = 1.37 \times 3.0 \times 0.00465$ = 0.0191 kg/m^2-s .

Soln (Contd.) - 1 - L39($\frac{8}{16}$)

Now, $p_{v,\infty} = 0.9 \times p_{sat}\,(27^0 C) = 0.9 \times 0.03567$ = 0.0321 bar and $\omega_{v,\infty}$ = 0.0321 /(1.61 $\times$ 1 $-$ 0.61 $\times$ 0.0321) = 0.02018 .
Similarly, p_{sat} (82.5^0C) = 0.5261 bar and $\omega_{v,w}$ = 0.4081 .
Thus, B = (0.02018 - 0.4081)/ (0.4081 -1) = 0.6554 , and

$$\frac{g}{g^*} = \frac{\ln(1+B)}{B} \times \left(\frac{Pr}{Sc}\right)^{0.33} \times \left(\frac{M_{mix,w}}{M_{mix,\infty}}\right)^{0.667}$$

where $M_{mix,w}$ = 24.51 and $M_{mix,\infty}$ = 28.76. Therefore, substitution gives g = $g^* \times$ 0.7039 = 0.01344 kg/m^2-s .
Hence, evaporation rate is
$\dot{m} = g \times B \times A_{plate} = 0.01344 \times 0.6554 \times 0.33 \times 1$
= 0.0029 kg/s, or 10.46 kg/hr (Ans a) .

Soln (Contd.) - 2 - L39($\frac{9}{16}$)

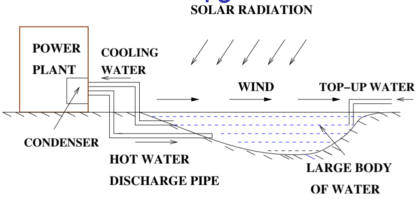
In this case, since Le $\simeq$ 1, we have

$$
\begin{aligned}
B &= \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{TL} + q_l/N_w} = 0.6554 \quad \text{Taking } T_{ref} = 0 \\
h_{m,\infty} &= 1.005 \times 27 + (0.875 \times 27 + 2503) \times 0.02018 \\
&= 78.12 \ \text{kJ/kg} \\
h_{m,w} &= 1.005 \times 82.5 + (0.875 \times 82.5 + 2503) \times 0.4081 \\
&= 1133.85 \ \text{kJ/kg} \\
h_{TL} &= 4.187 \times 82.5 = 345.4 \ \ \text{kJ/kg}
\end{aligned}
$$

Hence (q_l/N_w) = - 2400 kJ / kg. Negative sign indicates that heat is to be supplied. Thus,
$Q_{supp} = 2400 \times 0.0029 = 6.98$ kW (Ans b) .

Evaporation from a Lake - L39($\frac{10}{16}$)

Prob: A 10 kmph breeze (at 40^{0}C and 20 % RH) blows over a lake (at 30^{0}C). The lake receives q_{solar} = 500 $\frac{W}{m^2}$ Calculate the time required for the water level to drop by 1 cm . Assume turbulent boundary layer. Take $\nu = 15 \times 10^{-6}\ m^2/s$ Sc = 0.61 and Pr = 0.7.



Soln: Here

$p_{v,\infty} = 0.2 \times p_{sat}\,(40^0 C)$.

= 0.2×0.07384

= 0.01477 bar. Therefore,

$\omega_{v,\infty} = \frac{0.01477}{1.61 - 0.61 \times 0.01477}$

= 0.00919 .

Then, assuming Le $\simeq$ 1

$$B = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1} = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_{TL} + q_l/N_w}$$

where $\omega_{v,w}$, $h_{m,w}$ and q_l are not known.

Soln (Contd.) - 1 - L39($\frac{11}{16}$)

But, $N_w\, h_T + q_{rad} + q_l = N_w\, h_{TL}$. Hence,

$$B_h = \frac{h_{m,\infty} - h_{m,w}}{h_{m,w} - h_T - q_{rad}/N_w}$$

Now, $N_w = g \times B_h$. Hence

$$B_h = \frac{h_{m,\infty} - h_{m,w} + q_{rad}/g}{h_{m,w} - h_T} = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1}$$

where taking $T_{ref} = 0$

$$\begin{aligned} h_{m,\infty} &= 1.005 \times 40 + (0.875 \times 40 + 2503) \times 0.00919 \\ &= 63.3 \;\; \text{kJ/kg} \\ h_{m,w} &= 1.005\, T_w + (0.875\, T_w + 2503)\, \omega_{v,w} \\ h_T &= 4.187 \times 30 = 125.4 \;\; \text{kJ/kg} \end{aligned}$$

Soln (Contd.) - 2 - L39($\frac{12}{16}$)

Here, U_∞ = 10 kmph = 2.78 m/s. Hence,
$Re_L = 2.78 \times 100/15 \times 10^{-6}$ = 18.53×10^6
If we assume that $B \to 0$, then $g \to g^*$ and for a turbulent BL,

$$\frac{g^*}{\rho_m\, U_\infty} = 0.0365\, Re_L^{-0.2}\, Pr^{-0.67} = 0.001617$$

where $\rho_m = \rho_v + \rho_a = \rho_a\,(1 + W_{mean})$ and
$g^* = 0.00449\,\rho_m$ or $\frac{q_{rad}}{g^*} = \frac{0.5}{(0.00449\,\rho_m)} = (\frac{111.36}{\rho_m})$ kJ/kg .
Therefore, subtitution gives

$$\begin{aligned} B &= \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1} \\ &= \frac{63.3 - [1.005\, T_w + (0.875\, T_w + 2503)\,\omega_{v,w}] + (\frac{111.36}{\rho_m})}{[1.005\, T_w + (0.875\, T_w + 2503)\,\omega_{v,w}] - 125.4} \end{aligned}$$

where $\omega_{v,w}$ corresponds to T_w at stauration and ρ_m is evalutated from $W_{mean} = 0.5\,(W_w + W_\infty)$.

Soln (Contd.) - 3 - L39($\frac{13}{16}$)

We need trial-and-error procedure as follows.

1. Assume T_w. Hence, determine $p_{v,w}$ from steam tables and evaluate $W_w = 0.622 \times p_{v,w}/(p_{tot} - p_{v,w})$. Hence, evaluate W_{mean} and T_{mean}. Use gas law to evaluate, ρ_a at T_{mean} and, hence $\rho_m = \rho_a\,(1 + W_{mean})$
2. Substitute in the 2 eqns for B. If LHS = RHS, accept T_w and $\omega_{v,w}$.

In the present case, $T_w \simeq 39.5^0$C, $\omega_{v,w} = 0.043$ and ρ_m = 1.16 giving B = 0.0354 and $N_w = g^*\,B = 2.035 \times 10^{-4}$ kg / m^2-s, or 0.7324 kg / m^2-hr .

Now, for Δh = 1 cm drop in water level, we use mass balance $\rho_{water} \times (\Delta h/\Delta t) = N_w$.

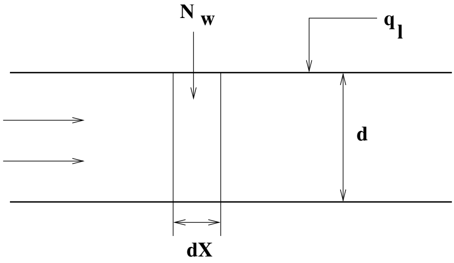
This gives Δt = 13.65 hrs (Ans) .

Note that the lake surface temperature is very cose to the free stream air temperature.

Humidification - L39($\frac{14}{16}$)

Prob: Moist air (W_{in} = 0.003 at 24^{0}C) enters a tube (2.5 cm dia, 75 cm long) at the rate of 9 kg /hr. The tube-wall is maintained wet at 24^{0}C. Calculate (a) W_{exit} and (b) Heat to be supplied to the tube wall. Take D = 0.09 m^2/hr and Sc = 0.6 Assume Temp remains constant.

Soln: Part (a) In this case $\omega_{v,w}$ is constant with x.



We define

$B_x = (\omega_{v,x} - \omega_{v,w})/(\omega_{v,w} - 1)$.

Then, $d\,\omega_{v,x} = (\omega_{v,w} - 1)\,d\,B_x$.

For the differntial element dx

$N_{w,x}\,dx = (\dot{m}/\pi\,d) \times d\omega_{v,x}$ or

$g\,B_x\,dx = (\dot{m}/\pi\,d)\,(\omega_{v,w} - 1)\,dB_x$

or, (next slide)

Soln (Contd.) - 1 - L39($\frac{15}{16}$)

$$\frac{d\,B_x}{B_x} = \left\{ \frac{\pi\, d\, g}{\dot{m}\,(\omega_{v,w} - 1)} \right\}\, dx \;\rightarrow\; \frac{B_{out}}{B_{in}} = \exp\left\{ \frac{\pi\, d\, g\, L}{\dot{m}\,(\omega_{v,w} - 1)} \right\}$$

where L = tube length. In this problem, corresponding to 24^0C, $\omega_{v,w}$ = 0.01875 (W_w = 0.01632) and corresponding to W_{in} = 0.003, $\omega_{v,in}$ = 0.002991.
Therefore B_{in} = 0.01606 .
Determination of g: At inlet, $W_m = 0.5\,(W_w + W_{in})$ = 0.01088 and at 24^0C, ρ_a = 1.189. Therefore $\rho_m = \rho_a\,(1 + W_m)$ = 1.202.
Further, $\nu_m = Sc \times D$ = 0.054 m^2/hr = 1.5 $\times$ 10^{-5} m^2/s.
Also, $\dot{m} = u_{in} \times (\pi/4)\, d^2$ or u_{in} = 5.09 m/s.
Hence, Re = $u_{in}\, d/\nu_m$ = 8488.3. Now, taking Le = 1,
$Sh = g\, d/(\rho_m\, D) = 0.023\, Re^{0.8}\, Sc^{0.4} = 26.06$
or, g = 112.76 kg/m^2-hr. Substitution gives B_{out} = 0.00757 .

Soln (Contd.) - 2 - L39($\frac{16}{16}$)

Hence, $\omega_{v,out} = \omega_{v,w} + B_{out}\,(\omega_{v,w} - 1) = 0.01137$
or W_{out} = 0.01145 kg / kg of dry air (Ans) .
Now Energy balance over element dx gives
$\dot{m} \times dh_{m,x} = (N_{w,x}\, h_{TL} - q_{l,x}) \times \pi\, d\, dx$. Integration gives

$$
\begin{aligned}
-\,\overline{q}_l &= -\,\frac{1}{L}\int_0^L q_{l,x}\,dx \\
&= (\frac{\dot{m}}{\pi\, d})\int_0^L \frac{d\,h_m}{dx}\,dx - g\,h_{TL}\int_0^L B_x\,dx \\
&= \frac{\dot{m}}{\pi\, d}\,[h_{m,out} - h_{m,in} - h_{TL}\,(\omega_{v,out} - \omega_{v,in})]
\end{aligned}
$$

where taking T_{ref} = 0, $h_{m,out}$ = 52.82, $h_{m,in}$ = 31.664
and $h_{TL} = 4.187 \times 24$ = 100.49 kJ/kg.
Hence $\overline{q}_l = -\,0.646\ \ kW/m^2$ (Ans)