ME-662 CONVECTIVE HEAT AND MASS TRANSFER

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LECTURE-31 CONVECTIVE MASS TRANSFER

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1. Importance
2. Main Task
3. Governing Transport Eqns
4. Boundary Layer Forms
5. Simplified Forms
    1. Stefan Flow Model
    2. Couette Flow Model
    3. Reynolds Flow Model

Importance of CHM L31($\frac{1}{14}$)

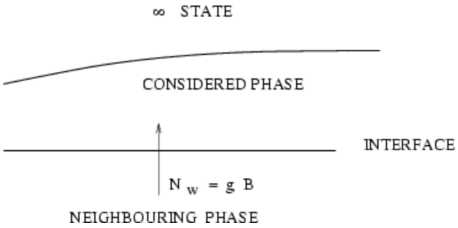
1. In several applications such as evaporative or transpiration cooling, combustion, ablation, condensation, drying etc , convective mass transfer assumes importance.
2. CHM takes place between two dissimilar phases such as liquid - gas or solid - gas. Hence, CHM is also referred to as Inter-phase Mass Transfer
3. In the absence of fluid motion, mass transfer takes place by diffusion only. Analogous to heat conduction , diffusion mass transfer takes place within a single phase ; gas, liquid or solid
4. Examples of diffusion mass transfer are: Leakage of hydrogen by diffusion through the solid container wall, Penetration of carbon through solid iron during hardening , diffusion of water vapour through solid during drying , diffusion of sugar through stagnant liquid tea in a cup

Main Task - L31($\frac{2}{14}$)

1. CHM takes place due to concentration gradients of the transferred species
2. The main task is to calculate Interface mass transfer flux N_w (kg/m^2-s) from

   $$N_w = g\,B$$

   where g (kg/m^2-s) is mass transfer coefficient and B is dimensionless driving force
   .
3. N_w and g have same units



N_w is positive when mass transfer takes place from the *neighbouring phase* into the *considered phase* across the interface & *vice versa*

Concentration defined - L31($\frac{3}{14}$)

1. Both neighboring and considered phases are mixtures comprising several species k. The proportion of k in the mixture can be defined in several ways.
2. Mass Fraction
$$\omega_k \equiv \frac{m_k}{m_{mix}} = \frac{\rho_k}{\rho_{mix}}$$
3. Mole Fraction
$$x_k \equiv \frac{n_k}{n_{mix}} = \frac{p_k}{p_{mix}}$$
where n = number of moles and p_k is partial pressure.
4. Concentration
$$[k\,] \equiv \frac{\rho_k}{M_k} = \frac{p_k}{R_u\,T} = \frac{1}{\overline{v}_k} \qquad (\frac{kmol}{m^3})$$
where M_k is molecular weight.
5. Most often, we shall prefer mass fraction ω_k which represents proportion by mass.

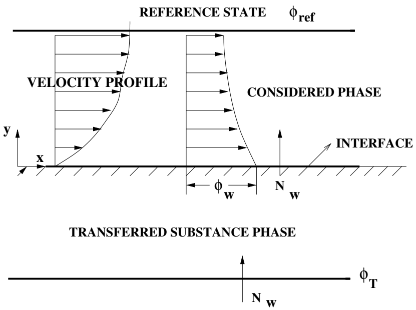
Driving force defined - L31($\frac{4}{14}$)

Unlike in heat transfer, in mass transfer, 3 states must be considered:

1. Reference state (ref) far into the *Considered Phase*
2. Interface state (w)
3. Trans Subs state (T) deep into *Neighbouring Phase*

Dimensionless Driving force B is

$$B = \frac{\Phi_{ref} - \Phi_w}{\Phi_w - \Phi_T}$$



where Φ is a Conserved property . Φ can be formed from ω_k, x_k or from [k].

Types of Mass Transfer - L31($\frac{5}{14}$)

In general, there are 3 types of mass transfer

1. *Mass Transfer* without heat transfer (no chemical reaction)
2. *Mass Transfer* with heat transfer (no chemical reaction)
3. *Mass Transfer* with heat transfer and chemical reaction

In each case, Conserved Property Φ must be appropriately defined. Any eqn of the form

$$\frac{\partial(\rho_m \Phi)}{\partial t} + \frac{\partial}{\partial x_j} \left[\rho_m u_j \Phi - \Gamma_\Phi \frac{\partial \Phi}{\partial x_j}\right] = S_\Phi$$

in which $S_\Phi = 0$, Φ is called a conserved property.

Transport Eqns - L31($\frac{6}{14}$)

From lectures 2 and 3

$$\frac{\partial(\rho_m)}{\partial t} + \frac{\partial(\rho_m\, u_j)}{\partial x_j} = 0 \quad \text{(Bulk mass)}$$

$$\rho_m \frac{D\, u_i}{D\, t} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu_{m,eff}\,\frac{\partial u_i}{\partial x_j}\right] + \rho_m\, B_i + \frac{\partial}{\partial x_j}\left[\mu_{m,eff}\,\frac{\partial u_j}{\partial x_i}\right] \quad \text{(Momentum)}$$

$$\rho_m \frac{D\, \omega_k}{D\, t} = \frac{\partial}{\partial x_j}\left(\rho_m\, D_{eff}\,\frac{\partial \omega_k}{\partial x_j}\right) + R_k \quad \text{(Species Transfer)}$$

$$\rho_m \frac{D\, h_m}{D\, t} = \frac{\partial}{\partial x_j}\left[k_{m,eff}\,\frac{\partial T}{\partial x_j}\right] - \frac{\partial(\sum m''_{j,k}\, h_k)}{\partial x_j} + \frac{D\, p}{D\, t} + \dot{Q}_{chem} + \dot{Q}_{rad} + \mu_{eff}\,\Phi_v \quad \text{(Energy)}$$

Boundary Layer Model - L31($\frac{7}{14}$)

From Near-interface considerations

$$\frac{\partial(\rho_m \, \Psi)}{\partial t} + \frac{\partial(\rho_m \, u \, \Psi)}{\partial x} + \frac{\partial(\rho_m \, v \, \Psi)}{\partial y} = \frac{\partial}{\partial y} \left[\Gamma_\Psi \, \frac{\partial \Psi}{\partial y} \right] + S_\Psi$$

Ψ	Γ_Ψ	S_Ψ
1	0	0
u	$\mu_{m,eff}$	$- \, dp \, / \, dx + B_x$
ω_k	$\rho_m \, D_{eff}$	R_k
h_m	$k_{m,eff} / cp_m$	$Dp/Dt + \dot{Q}_{rad} + \dot{Q}_{others}$ $-\partial(\sum m''_{y,k} \, h_k)/\partial y + \mu_{eff} \, (\partial u / \partial y)^2$

where $m''_{y,k} = - \, \rho_m \, D_{eff} \, \partial\omega_k / \partial y$.

Comments on BL model - L31($\frac{8}{14}$)

1. In the above eqns, when chemical reaction is present, species enthalpy is defined as

$$h_k(T) \equiv h^0_{f,k} + \Delta h_{s,k} = h^0_{f,k} + \int_{T_{ref}}^{T} c_{p,k}\, dT$$

where $h^0_{f,k}$ is Enthalpy of formation .

2. Also, from Fick's law of mass diffusion

$$m''_{y,k} = -\, D_{eff}\, \frac{\partial \rho_k}{\partial y} = -\, \rho_m\, D_{eff}\, \frac{\partial \omega_k}{\partial y}$$

3. Bl model is an idealisation of the transport eqns . It still involves simultaneous soln of several coupled differential eqns which requires computer solutions.
4. To avoid this, we shall postulate simpler models to serve a limited purpose but the solutions from model eqns can reveal the tendencies inherent in the complete BL model.

Element Transport Eqn - 1 - L31($\frac{9}{14}$)

1. Chemical reactions obey the *element conservation principle* . Thus, consider a mixture comprising CH_4, O_2, H_2, H_2O, CO_2, CO, N_2, NO, and O.
2. Then, the mass fraction (symbol η) of elements C, H, O, and N in the mixture will be related to ω_k as

$$\begin{aligned}
\eta_C &= \frac{12}{16}\,\omega_{CH_4} + \frac{12}{44}\,\omega_{CO_2} + \frac{12}{28}\,\omega_{CO}\ , \\
\eta_H &= \frac{4}{16}\,\omega_{CH_4} + \frac{2}{2}\,\omega_{H_2} + \frac{2}{18}\,\omega_{H_2O}\ , \\
\eta_O &= \frac{32}{44}\,\omega_{CO_2} + \frac{16}{28}\,\omega_{CO} + \frac{32}{32}\,\omega_{O_2} + \frac{16}{18}\,\omega_{H_2O} \\
&+ \frac{16}{16}\,\omega_{O} + \frac{16}{30}\,\omega_{NO} \quad \text{and} \quad \eta_N = \frac{14}{30}\,\omega_{NO} + \frac{28}{28}\,\omega_{N_2}
\end{aligned}$$

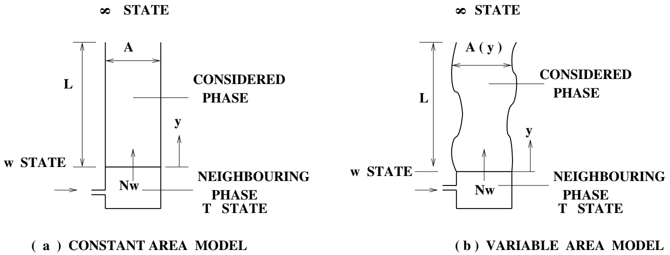
Element Transport Eqn - 2 - L31($\frac{10}{14}$)

1. In general, therefore, the *mass fraction* η_α of element α is $\eta_\alpha = \sum_k \eta_{\alpha,k}\, \omega_k$ where $\eta_{\alpha,k} = M_\alpha / M_k$ is the mass fraction of element α in the species k.
2. Just as the species are convected, diffused, and generated or destroyed, the elements can also be considered to have been convected and diffused, but *they can never be destroyed or generated* because of the principle of element conservation.
3. Thus, the transport equation for any element α will have no source term.

$$\frac{\partial(\rho_m\, \eta_\alpha)}{\partial t} + \frac{\partial(\rho_m\, u_j\, \eta_\alpha)}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\rho_m\, D\, \frac{\partial \eta_\alpha}{\partial x_j}\right)$$

where it is assumed that D for the elements is same as that for the species. η_α is always a conserved property.

Stefan Flow Model - L31($\frac{11}{14}$)

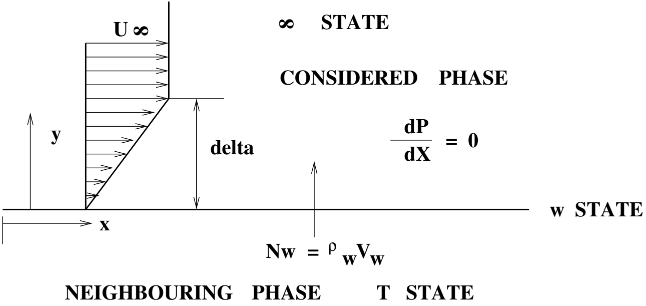


In this 1D model, we set u = dp/dx = 0 and A = A (y). Then, allowing for area change in y-direction, the BL model Eqn will transform to

$$\frac{d}{dy}\,[N_{\Psi,y}\,A] = \frac{d}{dy}\left[\rho_m\,v\,A\,\Psi - \Gamma_\Psi\,A\,\frac{d\,\Psi}{dy}\right] = A\,S_\Psi$$

where $\mu_{m,t} = D_t = k_{m,t} = 0$ Laminar diffusion only .

Couette Flow Model - L31($\frac{12}{14}$)



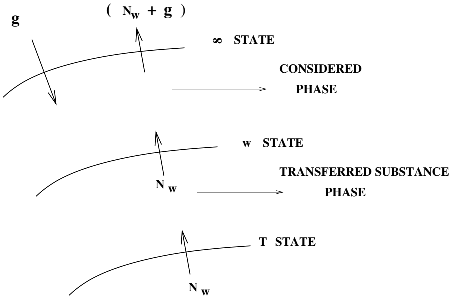
In this 1D model, we again set dp/dx = 0, but $u = C \times y$, $\partial / \partial x = 0$ and A = const. Then, in this idealisation, BL model reads as

$$\frac{d}{dy} \; [N_{\Psi,y}] = \frac{d}{dy} \; \left[\rho_m \, v \, \Psi - \Gamma_\Psi \, \frac{d \; \Psi}{dy} \right] = S_\Psi$$

This model permits study of effect of turbulent $\Gamma_{\Psi,t}$ and of property variations

Reynolds Flow Model - L31($\frac{13}{14}$)

1. This is an Algebraic model[a] - no differential eqn
2. The model postulates inward flux g at imaginary surface in the ∞ state carrying with it properties of ∞-state and an Outward flux (N_w + g), at the same imaginary surface, carrying with it properties of the w-state



Thus, there is no net mass creation between w and ∞ states. The model claims to account for all the effects produced at the *w surface in a real boundary layer model*.

[a] Spalding D B Convective Mass Transfer, Edward Arnold Ltd, London (1963)

Comment on the models - L31($\frac{14}{14}$)

1. Like in heat transfer, the greater part of the resistance to mass transfer is confined to near-wall region.
2. Hence, boundary layer flow model should suffice. But, this requires computer solutions of the governing eqns.
3. To avoid this, and to establish validity of the $N_w = g \times B$ simplified models are invoked.
4. Stefan flow and Couette flow models are One-dimensional and hence, analytically derived *closed form* solutions are possible.
5. Reynolds flow model is the simplest, being algebraic.
6. In the lectures to follow, the model solutions will be developed for the 3-types of mass transfer