ME-662 CONVECTIVE HEAT AND MASS TRANSFER

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LECTURE-28 TURBULENCE MODELS-3

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1. Laminar-to-Turbulent Transition
2. Effect of Rough Wall
3. Heat Transfer near a wall

What is Transition L28($\frac{1}{15}$)

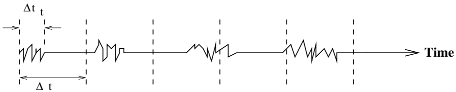
1. When Re_{D_h} in a duct or Re_x in a BL is increased, laminar flow undergoes *transition* before turning into fully turbulent one.
2. The transition phenomenon occurs over a range of Re - the lower limit indicates end of laminar conditions whereas the upper limit signifies establishment of fully turbulent conditions
3. Both f and Nu demonstrate unique features. Many engineering equipment operate in the transition regime. For example, Heat exchangers in process industries . On a gas-turbine blade, the pr gr varies in the range $-10^{-8} < \nu/U_\infty^2\,(\partial U_\infty/\partial y) < 10^{-5}$ and free-stream turbulence intensity in the range 2 % $<$ Tu $<$ 10 %. In these conditions, transition range of Re_x may well occupy as much as 50 % of the chord length.

Intermittancy - Pipe flow - L28($\frac{2}{15}$)

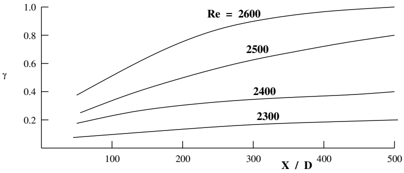
In a pipe flow, when $Re_D > 2300$, f increases with Re and u-vel at a fixed point shows periodic laminar and turbulent bursts. The latter do not occur at equal Δt . Intermittancy is defined as

$$0 < \gamma = N^{-1} \left[\sum_{k=1}^{N} \left(\frac{\Delta t_t}{\Delta t} \right)_k \right] < 1$$

γ varies with r and x - usually cross-sectional average is considered. In practical flows, $\gamma = 1$ is reached for x/D < 100.



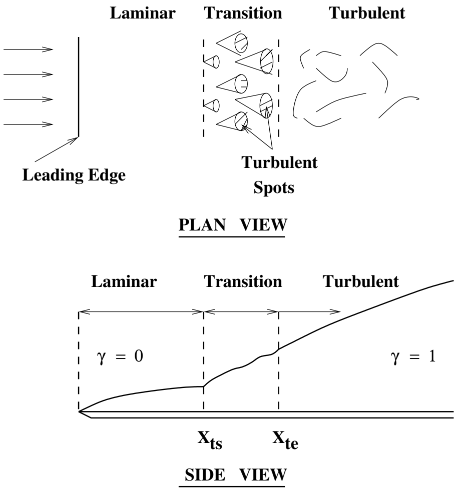
Axial vel in a pipe



Axial variation of intermittancy

Fully turbulent flow conditions at $Re_D \simeq 5000$

Intermittancy - Ext BL - L28($\frac{3}{15}$)



Transition is identified with occurance of intermittent turbulent spots surrounded by laminar fluid

Modeling γ and x_{ts} - Eqn - L28($\frac{4}{15}$)

1. Intermittancy variation is given by

$$\gamma = 1 - \exp(-412\,\beta^2), \quad \beta = \frac{(x - x_{ts})}{(x_{\gamma=0.75} - x_{\gamma=0.25})} \quad \text{(Narasimh}$$

$$\gamma = 1 - \exp(-5\,\xi^3) \quad \xi = \frac{(x - x_{ts})}{(x_{te} - x_{ts})} \quad \text{(Abu-Ghanam)}$$

2. At x_{ts} sudden departure in laminar variation of δ, C_{fx} or St_x.

$$Re_{\delta_{2,s}} = 1.174 \left[1 + \frac{22400}{Re_{x_{ts}}}\right] Re_{x_{ts}}^{0.46} \quad \text{(Cebeci)}$$

$$Re_{\delta_{2,s}} = 163 + \exp\left[(1 - \frac{Tu}{6.91})\, f(m)\right] \quad \text{(Fraser)}$$

$$f(m) = 6.91 - 12.75\,m + 63.64\,m^2 \quad m \geq 0,$$

$$f(m) = 6.91 - 2.48\,m - 12.27\,m^2 \quad m \leq 0$$

$$\text{and } m = -(\delta_2^2/\nu)\,(dU_\infty/dx) \qquad Tu = \sqrt{\overline{u'^2}}/U_\infty$$

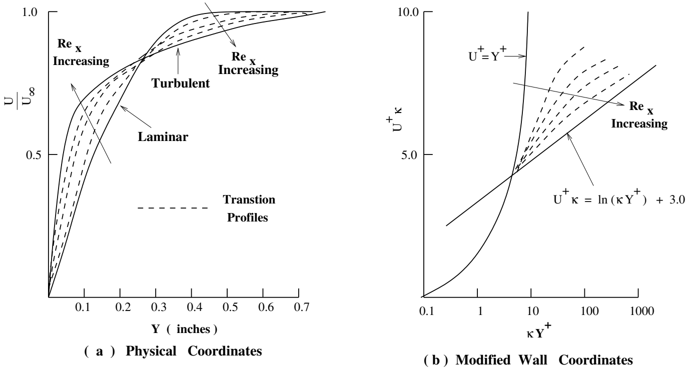
Modeling x_{te} - L28($\frac{5}{15}$)

End of Transition is estimated[1]

$$
\begin{aligned}
\frac{Re_\sigma}{Re_{\sigma_o}} &= 4.6 \left[1 + 1710\, m^{1.4} \, \exp\left\{-\left(1 + Tu^{3.5}\right)^{0.5}\right\}\right]^{-1} \text{ (FM)} \\
Re_\sigma &= \frac{U_\infty \left(x_{te} - x_{ts}\right)}{\nu} \quad Re_{\sigma_o} = \frac{\left(2.7 - 2.5\, Tu^{3.5}\right)}{\left(1 + Tu^{3.5}\right)} \times 10^5 \\
Re_\sigma &= 60\, Re_{xts}^{2/3} \text{ (CS)} \\
Re_{\delta_{2,xte}} &= 540 + 183.5 \left\{1.68 \times 10^{-4}\, Re_{x_{ts}}^{0.8} - 1.5\right\} \\
&\times \left(1 + m_{x_{ts}}\right) \text{ (DZ)}
\end{aligned}
$$

[1] **Fraser C. J. and Milne J. S.** *Integral Calculations of Transitional Boundary Layers*, Proc. Inst. Mech. Engrs., vol. 200, no: C3, p 179-187, 1986, **Cebeci T. and Smith A. M. O.** *Analysis of Turbulent Boundary Layers*, Academic Press, London, 1974, **Deutch and Zierke** , *The Measurement of Boundary Layers on a Compressible Blade in a Cascade*, NASA rep No 18511, Penn-State Univ, 1989

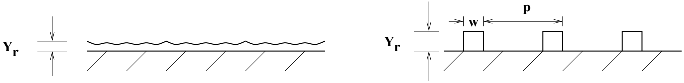
Profiles dp/dx = 0 - L28($\frac{6}{15}$)



$$\left\{\frac{u}{U_\infty}\right\}_{tr} = (1-\gamma)\left\{\frac{u}{U_\infty}\right\}_{lam} + \gamma\left\{\frac{u}{U_\infty}\right\}_{turb}$$

Effect of Wall-Roughness - L28($\frac{7}{15}$)

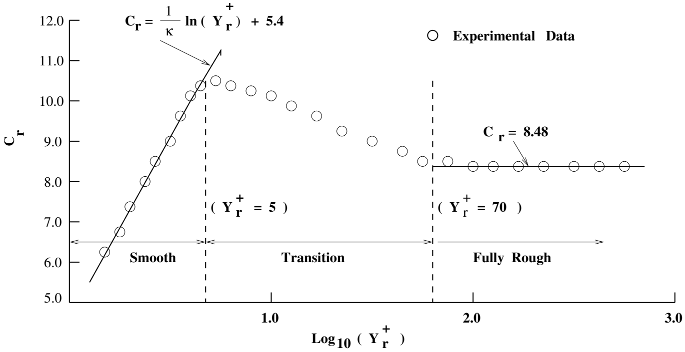
1. Smooth pipes have $(y_r/D) < 0.001$, where y_r is roughness height
2. Sometimes roughness is deliberately structured as in the form of square or triangular ribs to enhance heat transfer



(a) Continuous Sand Grain Roughness (b) Discrete Roughness

3. Near-wall law $\rightarrow u^+ = F(y^+, y_r^+) \rightarrow u^+ = \frac{1}{\kappa}\ln(y^+) + C(y_r^+)$ - There is no laminar sub-layer when $y_r^+ > 70$

Variation of $C_r \sim y_r^+$ - L28($\frac{8}{15}$)



Sand Grain Roughness by Nikuradze[2]

[2] Schlichting H. *Boundary Layer Theory*, Sixth edition, McGraw-Hill , New York, 1968

Discrete Roughness - L28($\frac{9}{15}$)

1. For discrete roughness, *equivalent* sand grain roughness (y_{re}) is defined.

$$
\begin{aligned}
u^+ &= \frac{1}{\kappa}\ln(\frac{y}{y_r}) + C_r' \\
y_{re} &\equiv y_r \exp\left[\kappa\,(8.48 - C_r')\right]
\end{aligned}
$$

   where C_r'(w, pitch) is determined experimentally.

2. Mixing length for rough surface is defined as

$$
\begin{aligned}
l_m &= \kappa\,(y + \Delta y_o)\left[1 - \exp(-\frac{y^+ + \Delta y_o^+}{A^+})\right] \\
\Delta y_o^+ &= 0.9\left[\sqrt{y_{re}^+} - y_{re}^+\exp\left(-y_{re}^+/6\right)\right] \quad 0 < y_{re}^+ < 70 \\
&= 0.7\,(y_{re}^+)^{0.58} \quad 70 < y_{re}^+ < 2000
\end{aligned}
$$

Near-wall Heat Transfer-1 - L28($\frac{10}{15}$)

1. From phenomenology $T - T_w = F\,(\,y,\ \tau_w,\ \mu,\ \rho,\ q_w, C_p, k)$
2. Therefore, dimensional analysis gives

$$
\begin{aligned}
T^+ &= F\,(\,y^+,\ Pr,\ \left[q_w^+\right]^{-1}\,) \text{ where} \\
T^+ &= \frac{-\,(T - T_w)}{q_w/(\rho\, C_p\, u_\tau)} \\
q_w^+ &= \frac{q_w}{\rho\, u_\tau^3} \quad (\text{ usually very small })
\end{aligned}
$$

Modeling $T^+ = F\,(\,y^+,\ Pr)$ (next slide)

Near-wall T^+ Law - L28($\frac{11}{15}$)

1. In the Inner layer , conv = 0. Then

$$-\frac{\partial}{\partial y}\left[\alpha\,\frac{\partial T}{\partial y}+\overline{v'\;T'}\right] = 0 \quad \text{or}$$
$$\frac{q_{tot}}{\rho\,C_p}=\frac{q_w}{\rho\,C_p} = -\left[\alpha\,\frac{\partial T}{\partial y}+\overline{v'\;T'}\right]$$

2. In the sub-layer

$$\frac{q_w}{\rho\,C_p}=-\,\alpha\,\frac{\partial T}{\partial y} \quad \text{or} \quad T^+ = Pr\,y^+ = Pr\,u^+$$

3. In the turbulent-layer ($max\ [30,\,30\,Pr] < y^+ < \delta^+$)

$$\frac{\partial T}{\partial y}=\frac{q_w}{\rho\,C_p\,u_\tau}\,\frac{u_\tau}{\nu}\,\frac{\partial F}{\partial y}$$

Independence from $\nu \rightarrow \partial F/\partial y^+ = \partial T^+/\partial y^+ = 1/(\kappa_T\,y^+)$
Or $T^+ = (1/\kappa_T)\ \ln(y^+) + C_T\,(\,Pr\,)$ where $\kappa_T \simeq \kappa/0.9 \simeq 0.44$

Continuous T^+ Law - 1 - L28($\frac{12}{15}$)

1. Previous slide shows that the T^+ Law can be generalised as $T^+ = \sigma \, \{u^+ + PF\}$ where in the laminar sublayer (LSL), σ = Pr and PF = 0.
2. Region between LSL and TSL is ill defined and In the turbulent layer (TSL)

$$T^+ = \frac{\kappa}{\kappa_T}\,(u^+ - 5.4) + C_T\,(Pr)$$

$$\sigma = \frac{\kappa}{\kappa_T} = Pr_T \ \ \text{(say)} \qquad PF = \frac{C_T\,(Pr)}{Pr_T} - 5.4$$

3. Therefore, for the entire inner layer

$$\tau_{tot} = \tau_w = \mu_{eff}\,\frac{\partial u}{\partial y} \quad q_{tot} = q_w = -k_{eff}\,\frac{\partial T}{\partial y} \ \ \text{gives}$$

$$Pr_{eff} = C_p\,\frac{\mu_{eff}}{k_{eff}} = \frac{\partial T^+}{\partial u^+} \ \ \text{or} \ \ T^+ = \int_0^\infty Pr_{eff}\, d\,u^+$$

Continuous T^+ Law - 2 - L28($\frac{13}{15}$)

1. Comparison with the generalised law shows that

$$PF = \int_0^{u^+} \left(\frac{Pr_{eff}}{Pr_T} - 1\right) d\, u^+ \;\text{ where}$$

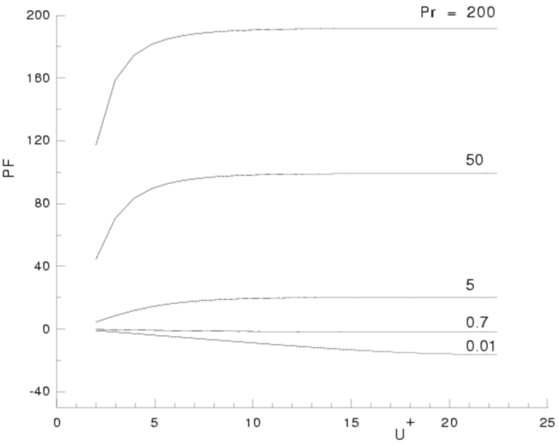
$$Pr_{eff} = \frac{\partial T^+/\partial y^+}{\partial u^+/\partial y^+} = Pr \left[\frac{1+\nu_t/\nu}{1+\alpha_t/\alpha}\right] \;\text{ and}$$

$$\frac{\nu_t}{\nu} = \left\{\left(\frac{\partial u^+}{\partial y^+}\right)^{-1} - 1\right\} \;\text{ and }\; \alpha_t = \frac{\nu_t}{Pr_T}$$

2. in the sub-layer, $\nu_t/\nu = 0$ and $Pr_{eff} = Pr$
3. $\partial u^+/\partial y^+$ can be obtained from the three-layer law or from continuous law from Van-Driest Mixing length allowing for effects of pr gr and suction/blowing.
4. PF can be interpreted as resistance to heat transfer in excess of that for momentum transfer . PF = 0, for Pr = 1

Continuous T^+ Law - 3 - L28($\frac{14}{15}$)

Numerical integration for smooth wall and $v_w = 0$ gives



Correlations for PF_∞ - L28($\frac{15}{15}$)

1. For Smooth Surfaces ,

$$PF_\infty = 9.24 \left[(\frac{Pr}{Pr_T})^{3/4} - 1 \right] \left[1 + 0.28 \, \exp(-0.007 \, \frac{Pr}{Pr_T}) \right]$$

$$PF_\infty = Pr_T \, (\frac{C_T \, (Pr)}{Pr_T} - 5.4) \quad \text{where}$$

$$C_T = \left[3.85 \, Pr^{1/3} - 1.3 \right]^2 + \kappa_T^{-1} \, \ln(Pr) \quad 0.006 < \mathrm{Pr} < 40000$$

2. For Rough Surfaces ,[3]

$$PF_{\infty,r} = 5.19 \, Pr^{0.44} \, {y_{re}^+}^{0.2} - 8.48 \;\; \text{(DS)}$$

$$PF_{\infty,r} = A \, Pr^{0.695} \, {y_{re}^+}^{0.395} \;\; \text{(Jay)} \rightarrow \text{A = F (3D element)}$$

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[3] **Dippery D. F. and Sebersky R. H.** *Heat and Momentum Transfer in Smooth and Rough Tubes in Various Prandtl Number*, IJHMT, vol. 6, p 329-353, 1963, **Jayatillake C. L. V.** *The Influence of Prandtl Number and Surface Roughness on Resistance of the Laminar Sublayer to Momentum and Heat Transfer*, Prog. in Heat Mass Transfer- 1, 1969