# ME-662 CONVECTIVE HEAT AND MASS TRANSFER

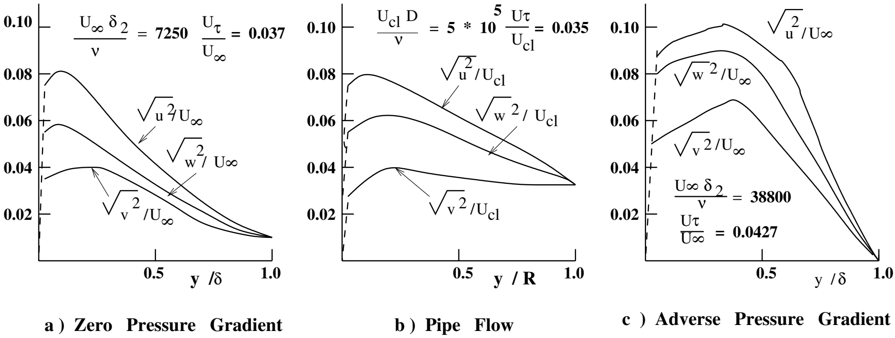
A. W. Date
Mechanical Engineering Department
Indian Institute of Technology, Bombay
Mumbai - 400076
India

## LECTURE-25 NEAR-WALL TURBULENT FLOWS-2

# LECTURE-25 NEAR-WALL TURBULENT FLOWS-2

1. Measured variation of turbulence quantities
2. Mean KE balance
3. Turbulent KE balance

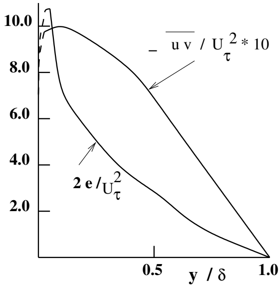
# Turbulence quantities - L25($\frac{1}{14}$)



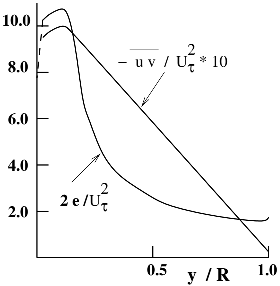
# Comments - L25($\frac{2}{14}$)

1. ( $\sqrt{u_i^2}$ ) in zero pr gr BL and pipe flow are similar. For an Adv pr gr BL, however, the intensities are greater and peak at a distance considerably away from the wall ( $y/\delta \simeq 0.35$).
2. ( $\sqrt{u^2}$ ) is highest and the ( $\sqrt{v^2}$ ) lowest in all cases. The outer layers thus demonstrate considerable anisotropy.
3. The position of the peak intensities suggests operation of a large turbulence production mechanism there.
4. The ( $e = u_i^2/2$ ) profiles are developed from the intensity data. ( see next slide )
5. $\sqrt{u_i^2}$ /e ( not shown ) over greater part of the outer layer is nearly const. This fact will be used later in the development of constants in the stress models.

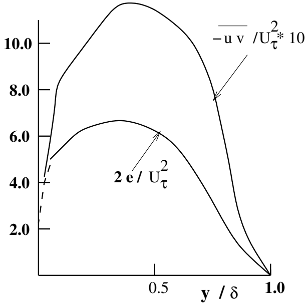
# Turbulent stress and KE - L25($\frac{3}{14}$)




( s ) Zero Pressure Gradient




( b ) Pipe Flow




( c ) Adverse Pressure Gradient

# Comments - L25($\frac{4}{14}$)

1. In fully developed pipe flow , $\partial u / \partial x = v = 0$ and dp/dx = const. Thus, $\tau_{tot} = \tau_l + \tau_t$ must vary linearly with radius. In the outer layer, $\tau_{tot} \simeq \tau_t$ and $\tau_t = -\rho \overline{uv}$ is indeed linear.
2. In zero pr gr BL , $\tau_t$ is nearly constant in the vicinity of $y/\delta \simeq 0.1$. Recall that this fact was used in the development of the logarithmic law for the inner region. It is remarkable that inspite of linear variation of $\tau_t = \tau_{tot}$ in the outer layers, the logarithmic law based on $\tau_{tot} = \tau_w$ = const assumption should have predicted the velocity profile upto nearly 50 to 60 percent width or upto $y^+ \simeq 700$.
3. In strongly Adv pr gr BL , the assumption $\tau_{tot}$ = const is not at all verified; as such, it was not possible to correlate the velocity profile by the logarithmic law beyond $y^+ = 100$
4. The outer layers are thus considerably influenced by the *history* as well as the boundary conditions.

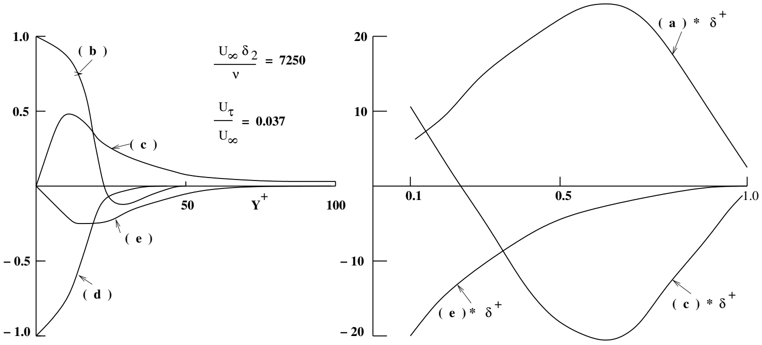
# Mean KE Eqn - Zero pr gr - L25($\frac{5}{14}$)

In order to study the mean KE balance, we non-dimensionalise MKE.

$$
\begin{aligned}
\underset{(a)}{\left[u^{+}\frac{\partial E^{+}}{\partial x^{+}} + v^{+}\frac{\partial E^{+}}{\partial y^{+}}\right]} &= \underset{(b)}{-\frac{\partial}{\partial y^{+}}\left(u^{+}\frac{\partial u^{+}}{\partial y^{+}}\right)} - \underset{(c)}{\frac{\partial}{\partial y^{+}}\left(u^{+}\frac{\overline{u'v'}}{u_\tau^2}\right)} \\
&- \underset{(d)}{\left(\frac{\partial u^{+}}{\partial y^{+}}\right)^2} - \underset{(e)}{\left(-\frac{\overline{u'v'}}{u_\tau^2}\frac{\partial u^{+}}{\partial y^{+}}\right)}
\end{aligned}
$$

where, $E^{+} = E/u_\tau^2$. Term ( a ) - Convection, ( b ) laminar diffusion, ( c ) turbulent diffusion, ( d ) viscous dissipation and ( e ) loss to turbulent KE production

# MKE Balance - Zero pr gr - L25($\frac{6}{14}$)




( a ) Inner Layer

( b ) Outer Layer

# Comments on MKE Balance - 1 - L25($\frac{7}{14}$)

Inner layer ( left fig )

1. In the inner 10 to 15 percent layer ( $y^+ < 100$ say ), convection (term a) is almost zero.
2. In the viscous sublayer( $y^+ \leq 5$ ), turbulent fluctuations almost vanish; as such laminar diffusion (term b) equals viscous dissipation (term d).
3. In the transition layer( $5 < y^+ < 30$ ), all terms on the right hand side are significant.
4. Viscous effects are negligible in the turbulent inner layer( $y^+ > 30$ ); hence, terms b and d are zero and, turbulent diffusion (term c) equals (term e)
5. When the Eqn is integrated from $y = 0$ to $y = \delta$, contribution of ( terms b and c ) vanishes. These terms simply redistribute energy within the boundary layer.

# Comments on MKE Balance - 2 - L25($\frac{8}{14}$)

Outer layer ( Right fig )

1. In the outer parts , the loss of mean energy to turbulence (term e) reduces with distance from the wall.
2. The gain in mean energy due to convection is practically due to the work done by the turbulent stresses (term c).
3. Near $y/\delta \simeq 0.1$, the stress terms, also make up for the loss to turbulence since convection , being small, is unable to compensate the loss.
4. There is thus a flux of energy from the outer layers to the inner wall region due to diffusion.

We shall now develop Turbulent KE balance.

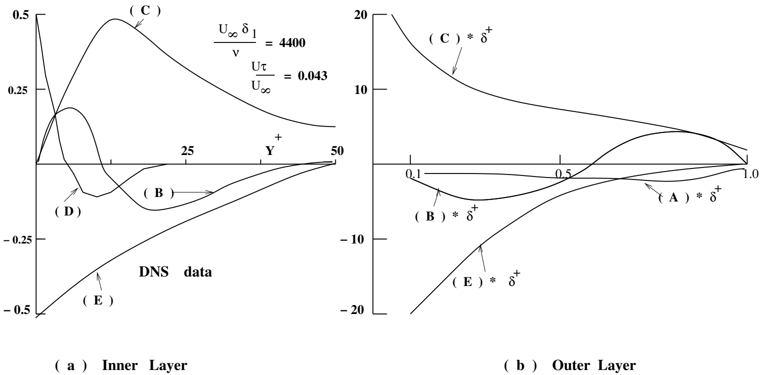
# Turbulent KE Eqn - Zero pr gr - L25($\frac{9}{14}$)

$$\underbrace{\left[u^+\frac{\partial e^+}{\partial x^+} + v^+\frac{\partial e^+}{\partial y^+}\right]}_{(A)} = -\underbrace{\left[\frac{\partial}{\partial y^+}\left(\frac{\overline{p'\,v'}}{\rho\,u_\tau^3}\right) + \frac{\partial}{\partial y^+}\left(\frac{\overline{e^+\,v'}}{u_\tau}\right)\right]}_{(B)}$$

$$+ \underbrace{\left(-\frac{\overline{u'v'}}{u_\tau^2}\,\frac{\partial u^+}{\partial y^+}\right)}_{(C)} + \underbrace{\frac{\partial^2 e^+}{\partial {y^+}^2}}_{(D)} - \underbrace{\frac{1}{u_\tau^2}\overline{\left(\frac{\partial u_j'}{\partial x_i^+}\right)^2}}_{(E)}$$

where, $e^+ = e/\,u_\tau^2$. Term ( A ) - Convection, ( B ) turbulent diffusion due to pr. and vel. fluctuations, ( C ) gain due to turbulent KE production, ( D ) Laminar diffusion and ( E ) loss due to Turbulent energy dissipation $\epsilon$.

# TKE Balance - Zero pr gr - L25($\frac{10}{14}$)



# Comments on TKE Balance - 1 - L25($\frac{11}{14}$)

Inner layer ( left fig - DNS data )

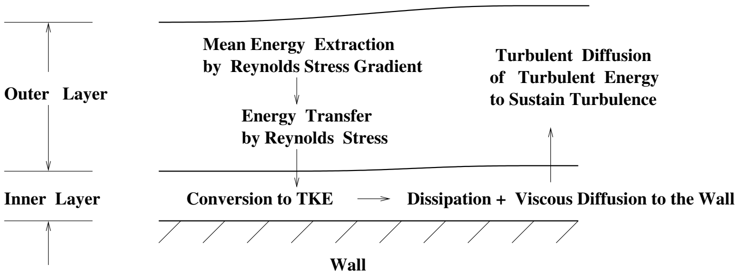
1. Terms B and D, again simply redistribute energy, their integrated value being zero.
2. There is high dissipation ( term E ) near the wall as expected
3. Production ( term C ) peaks at $y^+ \simeq 20$.

# Comments on TKE Balance - 2 - L25($\frac{12}{14}$)

Outer layer ( Right fig - Hot-wire data )

1. Unlike MKE convection, TKE convection ( term A ) remains almost negligible.
2. In the region of $y/\delta \simeq 0.1$, the energy production (term C) is accounted primarily by dissipation (term E) but, is also partly given to turbulent diffusion (term B) which transports the turbulent energy towards outer parts of the layer where it is dissipated.
3. Thus, a mechanism exists whereby there is an influx of MKE from the outer layers which is in part directly dissipated but in part diffused back by turbulence into the outer region.
4. It is this net pumping of energy that is responsible for the turbulent bursts at the wall and sustenance of turbulence in a turbulent flow.

# Net Energy Exch. - Zero pr gr - L25($\frac{13}{14}$)




The overall exchange mechanism between outer and inner regions. Such experimentally and DNS determined energy balances have been reported for both Adv. and Fav. dp/dx BLs as well as for pipe flow.

# Summary - L25($\frac{14}{14}$)

1. Measured variations of mean and turbulent quantities show the locations of dominant mechanisms.
2. The recent DNS computations have enhanced understanding of the processes in the inner layer and also have corroborated hot-wire measurements of the past
3. Understanding of the net exchange mechanism aids in development of turbulence models of different levels of complexity for both the outer layers as well as the viscosity affected inner layer.
4. Turbulence models are needed for both engineering flows as well as environmental/atmospheric flows.