

Graph Theory

A NPTEL Course

S.A. Choudum

Department of Mathematics

IIT Madras

Chennai, India

email: sac@iitm.ac.in

Notes to the Reader

- At a faster pace the course can be read in about 65 lectures and at a slower pace in about 100 lectures.
- By skipping certain topics (indicated as optional) the course can be read in about 45 lectures.
- Solve as many exercises as you can. Do not get bogged down on a single exercise for long hours. Revisit the exercises later.
- Those who would like to go for research are advised not to skip any topic.
- A list of the books/monographs is included. These books can be referred to for the proofs which we have not included.

Modules

1 Preliminaries (5 - 10 lectures)	1
1.1 Introduction: Discovery of graphs	2
1.2 Graphs	4
• Definitions	4
• Pictorial representation of a graph	4
• Isomorphic graphs	6
• Subgraphs	8
• Matrix representations of graphs	9
• Degree of a vertex	11
• Special graphs	13
• Complements	16
• Larger graphs from smaller graphs	16
Union	16
Sum	17
Cartesian Product	17
Composition	18
1.3 Graphic sequences	19
• Graph theoretic model of the LAN problem	20
• Havel-Hakimi criterion	21
• Realization of a graphic sequence	22

• Erdős-Gallai criterion	25
Exercises	28
2 Connected graphs and shortest paths (4-8 lectures)	33
2.1 Walks, trails, paths, cycles	34
2.2 Connected graphs	39
• Distance	43
• Cut-vertices and cut-edges	44
• Blocks	47
2.3 Connectivity	50
2.4 Weighted graphs and shortest paths	55
• Weighted graphs	56
• Dijkstra's shortest path algorithm	57
• Floyd-Warshall shortest path algorithm	61
Exercises	66
3 Trees (5 - 10 lectures)	71
3.1 Definitions and characterizations	72
3.2 Number of trees (Optional)	75
• Cayley's formula	77
• Kirchoff-matrix-tree theorem	79
3.3 Minimum spanning trees	83
• Kruskal's algorithm	84
• Prim's algorithm	88
Exercises	90

<i>MODULES</i>	iii
4 Special classes of graphs(6 - 12 lectures)	97
4.1 Bipartite Graphs	99
4.2 Line Graphs (Optional)	103
4.3 Chordal Graphs (Optional)	107
Exercises	114
5 Eulerian Graphs (2 - 4 lectures)	119
5.1 Motivation and origin	120
5.2 Fleury's algorithm	123
5.3 Chinese Postman problem (Optional)	128
Exercises	131
6 Hamilton Graphs (4 - 8 lectures)	135
6.1 Introduction	136
6.2 Necessary conditions and sufficient conditions	137
Exercises	146
7 Independent sets, coverings and matchings(8-16lectures)	151
7.1 Introduction	152
7.2 Independent sets and coverings: basic equations	152
7.3 Matchings in bipartite graphs	159
• Hall's Theorem	160
• König's Theorem	163
7.4 Perfect matchings in graphs	167
7.5 Greedy and approximation algorithms (Optional)	172

Exercises	176
8 Vertex Colorings (4 - 8 lectures)	179
8.1 Basic definitions	180
8.2 Cliques and chromatic number	182
• Mycielski's theorem	182
8.3 Greedy coloring algorithm	184
• Coloring of chordal graphs (Optional)	187
• Brooks theorem (Optional)	188
Exercises	191
9 Edge Colorings (8 - 16 lectures)	195
9.1 Introduction and Basics	196
9.2 Gupta-Vizing theorem	198
9.3 Class-1 and Class-2 graphs	201
• Edge-coloring of bipartite graphs	202
• Class-2 graphs	205
• Hajos union and Class-2 graphs (Optional)	208
9.4 A scheduling problem and equitable edge-coloring (Optional)	210
Exercises	214
10 Planar Graphs (10 - 20 lectures)	217
10.1 Basic concepts	218
10.2 Euler's formula and its consequences	223
10.3 Polyhedrons and planar graphs (Optional)	226

10.4	Characterizations of planar graphs	231
•	Subdivisions and Kuratowski's characterization	231
•	Minors and Wagner's theorem	241
10.5	Planarity testing (Optional)	242
•	D-M-P-planarity algorithm	243
10.6	5-Color-theorem	247
	Exercises	250
11	Directed Graphs (8 - 16 lectures)	255
11.1	Basic concepts	256
•	Underlying graph of a digraph	257
•	Out-degrees and in-degrees	258
•	Isomorphism	259
11.2	Directed walks, paths and cycles	259
•	Connectivity in digraphs	261
11.3	Orientation of a graph	265
11.4	Eulerian and Hamilton digraphs	268
•	Eulerian digraphs	268
•	Hamilton digraphs	269
11.5	Tournaments	272
	Exercises	278
	List of Books	283
•	<u>Old Classics</u>	283
•	<u>Text Books</u>	283

- Books on Selected Topics 284