Problem Sheet

Q1. Trace the sample path of the following stochastic processes:

(i) $\{W_k,\ k \in T\}$ where W_k be the time that the k^{th} customer has to wait before receiving service and $T = \{1, 2, \cdots\}$.

(ii) $\{X(t), t \in T\},\ X(t)$ being the number of jobs in system at time $t,\ T = \{t : 0 \leq t < \infty\}$.

(iii) $\{Y(t), t \in T\}$ where $Y(t)$ is cumulative service requirements of all jobs in system at time $t : 0 \leq t < \infty$.

Q2. Classify the following random processes according to state space and parameter space.

(i) Water level in a tank at time $t \geq 0$.

(ii) Number of customers in a shop at time $t \geq 0$.

(iii) Number of breakdowns of a machinery in each week.

(iv) Water level in tank at the end of each hour.

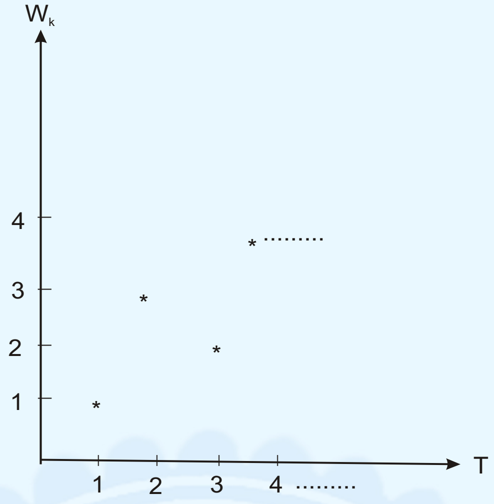
Q3. Give examples from real life situation which follow Poisson stochastic process. Specify parameter space and state space.

Q4. Give examples from real life situation which follow symmetric random walk. Specify parameter space and state space.

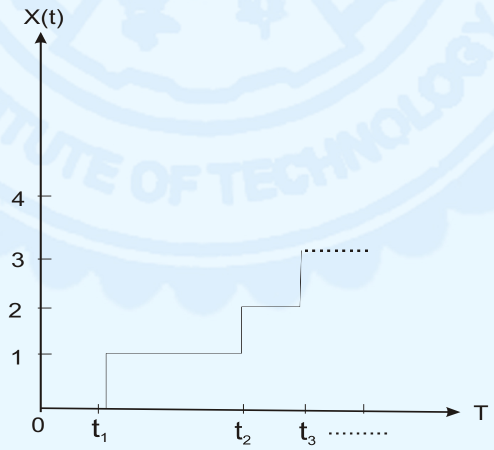
Solution to Problem Sheet

Ans 1. Sample paths:

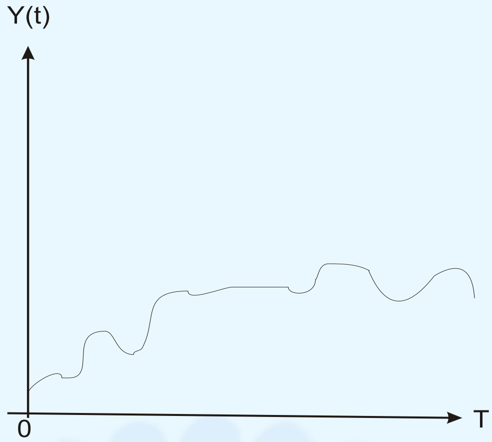
(i) Discrete time, Discrete space stochastic process.



(ii) Discrete state, continuous time stochastic process.



(iii) Continuous state, continuous time stochastic process



Ans 2. **(i)** Continuous state, continuous time stochastic process.

(ii) Discrete state, continuous time stochastic process.

(iii) Discrete state, discrete time stochastic process.

(iv) Continuous state, discrete time stochastic process.

Ans 3. Consider a coffee shop at which customers are arriving randomly. Let

X_n = no. of customers at the shop at the end of nth hour.

Then $\{X_n;\ n = 1, 2, 3, ..., 24\}$ is a Poisson process.

State space $S = \{1, 2, 3, ...\}$

Parameter space $T = \{1, 2, 3, ..., 24\}$.

Ans 4. Gambler ruin problem. Consider following

$$X_n = \text{out come of nth trial of the game.} = \begin{cases} 1 & \textit{with probability } \frac{1}{2} \\ -1 & \textit{with probability } \frac{1}{2}. \end{cases}$$

$$S_n = \text{wealth of gambler after nth trial} = \textstyle\sum_{i=1}^{n} X_i$$

State space $S = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$

Parameter space $T = \{1, 2, 3, ...\}$.