Module 2 : General Topology

Lecture 4 : Further preliminaries from general topology

Exercises

1. What happens if we omit the surjectivity hypothesis on the function $f : X \longrightarrow Y$ in the definition of quotient topology on Y induced by f ?
2. Show that the space obtained from the unit ball $\{\mathbf{x} \in \mathbb{R}^n / \|\mathbf{x}\| \leq 1\}$ by collapsing its boundary to a singleton, is homeomorphic to the sphere S^n.
3. Show that $\mathbb{R}P^1 \cong S^1$ by considering the map $f : S^1 \longrightarrow S^1$ given by $f(z) = z^2$.
4. Try to show that S^2 is not homeomorphic to $\mathbb{R}P^2$. Would the Jordan curve theorem help?
5. Show that the boundary of the Möbius band is homeomorphic to S^1.
6. Does a Möbius band result upon cutting the projective plane $\mathbb{R}P^n$ along a closed curve on it ?