Exercises:

1. Prove that any continuous function $f : [-1, 1] \longrightarrow [-1, 1]$ has a fixed point, that is to say, there exists a point $x \in [-1, 1]$ such that $f(x) = x$.
2. Prove that the unit interval $[0, 1]$ is connected. Is it true that if $f : [0, 1] \longrightarrow [0, 1]$ has connected graph then f is continuous? What if connectedness is replaced by path connectedness?
3. Suppose X is a locally compact, non-compact, connected Hausdorff space, is its one point compactification connected? What happens if X is already compact and Hausdorff?
4. Show that any connected metric space with more than one point must be uncountable. Hint: Use Tietze's extension theorem and the fact that the connected sets in the real line are intervals.
5. Show that the complement of a two dimensional linear subspace in $\mathbb{R}^4$ is connected. Hint: Denoting by V be the two dimensional vector space, show that $\Sigma = \{\mathbf{x}/\|\mathbf{x}\| \;/\; \mathbf{x} \in \mathbb{R}^4 - V\}$ is connected using stereographic projection or otherwise.
6. How many connected components are there in the complement of the cone

   $$x_1^2 + x_2^2 + x_3^3 - x_4^2 = 0$$

   in $\mathbb{R}^4$? Hint: The complement of this cone is filled up by families of hyperboloids. Examine if there is a connected set B meeting each member of a given family.
7. A map $f : X \longrightarrow Y$ is said to be a *local homeomorphism* if for $x \in X$ there exist neighborhoods U of x and V of $f(x)$ such that $f\Big|_U : U \longrightarrow V$ is a homeomorphism. If $f : X \longrightarrow Y$ is a local homeomorphism and a proper map, then for each $y \in Y$, $f^{-1}(y)$ is a finite set. Show that the map $f : \mathbb{C} - \{1, -1\} \longrightarrow \mathbb{C}$ given by $f(z) = z^3 - 3z$ is a local homeomorphism. Is it a proper map?