Exercises:

1. Prove that the map η in the five lemma is surjective.
2. Show that the map (38.2) is indeed an isomorphism. To prove that it is surjective use the decompositions $S^{\mathcal{U}}(X) = S(X - U) + S(\text{ int} A)$ and

   $S^{\mathcal{V}}(A) = S(A - U) + S(\text{ int} A)$
3. Prove the Barrett-Whitehead lemma.
4. Calculate the local homology groups $H_2(X, X - \{p\})$ in the following cases:

   (i)The space X is the cylinder $S^1 \times [0,1]$ and p a point on its boundary.

   (ii)The space X is the Möbius band and p is a point on its boundary.

   Deduce that the cylinder and the Möbius band are not homeomorphic.
5. A topological manifold is a Hausdorff space in which each point has a neighborhood homeomorphic to an open ball in $\mathbb{R}^n$. Show that if p is a point on a topological manifold M,

$$H_n(M,\ M - \{p\}) \cong \mathbb{Z}.$$