Module 7 : Relative homology, exicism and the Jordan Brouwer separation theorem Lecture 38 : Relative homology

## **Exercises:**

- 1. Verify that the diagram (37.5) commutes.
- 2. Determine  $H_n(X, A)$  when  $A = \emptyset$ , and when A is a singleton and  $n \ge 1$ . What happens if n = 0?
- 3. Compute  $H_n((S^1 \times S^1)/(S^1 \vee S^1))$  and compare it with the absolute homology  $H_n((S^1 \times S^1)/(S^1 \vee S^1))$ .
- 4. Compute  $H_k(E^n/S^{n-1})$  and compare it with  $H_k(E^n, S^{n-1})$ .
- 5. In example (35.1), prove that X/A is homeomorphic to  $\mathbb{R}P^2$ . Compare the groups

 $H_n(X, A)$  with the groups  $H_{n+1}(X, A)$ . Hint: To set up the homeomorphism note that  $(x, y) \mapsto (x\sqrt{1-y^2}, y)$  maps each  $[-1, 1] \times \{y\}$  homeomorphically onto the chord at height y.