

Module 6 : Basic homology theory

Lecture 32 :The abelianization of the fundamental group

**Exercises:**

1. Verify the displayed results for  $\partial\sigma_1$  and  $\partial\sigma_2$  in lemma (32.2).
2. By writing out the boundary formula in detail verify equations (32.5) and (32.6).
3. Prove lemma (32.5).
4. Verify the naturality of  $\Pi_X$  by proving that the diagram (32.1) commutes.
5. Determine the first homology group of the Klein's bottle.
6. Determine the first homology groups of all the spaces described in the exercises to lecture 26.