

Exercises:

1. Sketch Δ_n for $n = 1, 2, 3$. Show that Δ_n is a compact and connected subspace of \mathbb{R}^{n+1} .
2. Discuss the continuity of the maps (29.1). Prove lemma (29.1). what about the cases $i \leq j$?
3. Verify equation (29.8).
4. Determine the values of n ($n = 1, 2, \dots$) for which a constant function $\Delta_n \rightarrow X$ is an n -cycle.
5. Show that the family of all chain complexes forms a category in which the set of morphisms $\text{Mor}(G, K)$ between any two chain complexes G and K is the set of all chain maps from G to K .
6. Naturality of (29.17)-(29.18). Assume given a commutative diagram of chain complexes with exact rows:

$$\begin{array}{ccccccccc}
 0 & \longrightarrow & L & \xrightarrow{f} & G & \xrightarrow{g} & K & \longrightarrow & 0 \\
 & & \downarrow \phi & & \downarrow \psi & & \downarrow \eta & & \\
 0 & \longrightarrow & L' & \xrightarrow{f'} & G' & \xrightarrow{g'} & K' & \longrightarrow & 0
 \end{array}$$

Denoting by δ_n and δ'_n the connecting homomorphisms, sketch relevant diagrams and prove that

$$\delta'_n \circ H_n(\eta) = H_n(\psi) \circ \delta_n$$