**Exercises:**

1. Suppose that $G$ is a finite group acting freely on a Hausdorff space then the action is properly discontinuous and hence deduce that the group action in the example of the generalized Lens space is properly discontinuous.

2. Suppose that $p : \tilde{X} \longrightarrow X$ is a covering projection and $\tilde{X}$ is locally path connected and simply connected. Show that if $U$ is an evenly covered open set in $X$ and $\tilde{U}$ is a sheet lying above it then $\phi(\tilde{U}) \cap \tilde{U} = \emptyset$ for every $\phi \in$ Deck $(\tilde{X}, X)$ and $\phi \neq$ id $_{\tilde{X}}$. Deduce that the group of deck transformations acts properly discontinuously on $\tilde{X}$.

   How does this relate to theorem 17.2?

3. Does the fundamental group of Klein's bottle have elements of finite order? Identify this group with a familiar group that we have already encountered in lecture 7. What is its abelianization?

4. Show that the torus is obtained as the orbit space of a group of homeomorphisms acting properly discontinuously on $\mathbb{R}^2$. Write out these homeomorphisms explicitly.

5. Show that the torus is a double cover of the Klein's bottle. Hence the fundamental group of the Klein's bottle must contain a subgroup of index two. Determine this subgroup.

6. Show that the cylinder is a two-sheeted cover of the Möbius band.

7. Suppose that $G$ is a topological group, $H$ is a discrete subgroup of $G$. Show that there exists a neighborhood $U$ of the identity such that $U = U^{-1}$, $U \cap H = \{1\}$ and that $\{hU / h \in H\}$ is a family of disjoint open sets. Deduce that the quotient map $\eta : G \longrightarrow G/H$ is a covering projection. Also show that $G/h$ is Hausdorff.