Exercises:

1. For the map S in example (18.3) show that S_* is the map $\mathbb{Z} \longrightarrow \mathbb{Z}$ given by $x \mapsto 2x$.
2. Suppose G is a path connected topological group with unit element e and $p : \tilde{G} \longrightarrow G$ is a covering map. For any choice of $\tilde{e} \in p^{-1}(e)$ show that there is a group operation on $\tilde{G}$ with unit element $\tilde{e}$ that makes $\tilde{G}$ into a topological group and p is a continuous group homomorphism.
3. Show that if Ω is an open subset of $\mathbb{C} - \{0\}$ on which a continuous branch of the logarithm exists then this branch is automatically holomorphic. Likewise show that the continuous branch of $\sqrt{z(2z-1)}$ on $\mathbb{C} - [0, 1/2]$ obtained in the lecture is holomorphic.
4. Use the fact that S^{n-1} is not a retract of S^n to prove that $\mathbb{R}P^{n-1}$ is not a retract of $\mathbb{R}P^n$.
5. Show that any continuous map $S^n \longrightarrow S^1$ is homotopic to the constant map if $n \geq 2$. What about maps from the projective spaces $\mathbb{R}P^n \longrightarrow S^1$ ($n \geq 2$)?