**Exercises:**

1. Describe a path in $S^n$ whose image under the standard map represents the generator of $\pi_1(\mathbb{R}P^n, x_0)$.
2. Let $C_0$ be the unit circle in the complex plane and $\omega_1, \omega_2, \dots, \omega_n$ denote the $n-$th roots of unity and at each of these a circle $C_j$ of small radius touches the unit circle externally. Construct a continuous map $p$ from the union of these $n+1$ circles onto the figure eight loop such that $p$ is a regular covering. Hint: Take one lobe of the figure eight to be the unit circle $C_0$ and define $p(z) = z^n$ for $z \in C_0$. Let $L$ be the other lobe of figure eight touching the lobe $C_0$ at say the point $1$. For each $j$ let $p_j : C_j \longrightarrow L$ be any homeomorphism such that $p_j(\omega_j) = 1$. Use gluing lemma to glue these maps to obtain the desired covering. in
3. For the covering projection of the preceding exercise determine the Action of the fundamental group of the base on a fiber assuming that the loops $C_0$ and $L$ (based at $1$ ) generate the fundamental group.
4. Consider the covering projection of exercise 6, lecture 15. Show by studying the lifts of various loops based at $(1, 1)$ that the covering is regular. We shall see another proof of regularity of this covering in lecture 19.
5. For the covering considered in the preceding exercise, determine the lifts of the loops
$$\gamma_1 : t \mapsto 1 - \exp(2\pi i t), \quad \gamma_2 : t \mapsto -1 + \exp(2\pi i t).$$
Find the lift of $\gamma_1 * \gamma_2 * \gamma_1^{-1} * \gamma_2^{-1}$ and deduce that the fundamental group of the figure eight space is non-abelian.
6. Show that the figure eight loop $(S^1 \times \{1\}) \cup (\{1\} \times S^1)$ is not a retract of the torus $S^1 \times S^1$. Show that the figure eight loop is a deformation retract of the torus minus a point.