Exercises:

1. Use the general results of this section to give an efficient and transparent proof that $\pi_1(S^1, 1) = \mathbb{Z}$. First show that for any loop γ based at 1, the map $\pi_1(S^1, 1) \longrightarrow \mathbb{Z}$ given by $[\gamma] \mapsto \tilde{\gamma}(1)$ is well defined by theorem 16.1, is a group homomorphism using uniqueness of lifts. Show that surjectivity follows from uniqueness of lifts and injectivity follows from theorem 16.1.
2. Let X be a topological spaces and $a, b \in X$. A simple chain connecting a and b is a finite sequence $U_1, U_2, \dots, U_n$ of open sets such that $a \in U_1$, $b \in U_n$ and for $1 \leq i < j \leq n$, $U_i \cap U_j \neq \emptyset$ implies $j = i + 1$.

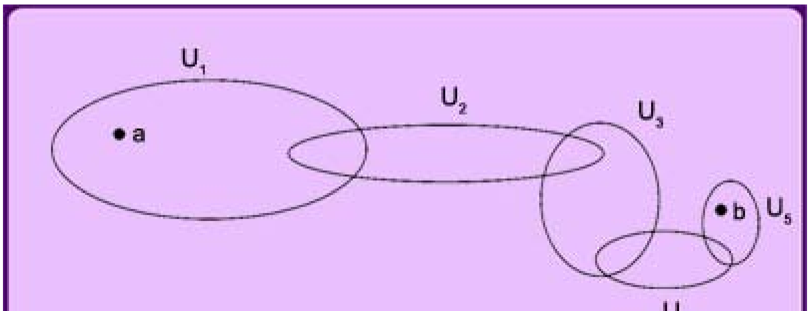

Figure 9 : Chain connectedness

Show that if X is a connected metric space and $\mathcal{U}$ is an open covering of X then any two points $a, b \in X$ can be connected by a simple chain. This property is referred to as chain connectedness. Is $\mathbb{Q}$ chain connected?
3. Use the above exercise to show that if X is a chain-connected space and $p : \tilde{X} \longrightarrow X$ is a covering projection then for any pair of points $x, y \in X$ the fibers $p^{-1}(x)$ and $p^{-1}(y)$ have the same cardinality. The point here is that X need not be path connected and the idea of using a path joining x and y as was done in the proof of theorem 14.4 is no longer available.
4. A toral knot is a group homomorphism $\kappa : S^1 \longrightarrow S^1 \times S^1$ given by $z \mapsto (z^m, z^n)$ where $m, n \in \mathbb{N}$. Regarding the toral knot as a loop on the torus determine its lifts with respect to the covering projection $\mathbb{R} \times \mathbb{R} \longrightarrow S^1 \times S^1$.
5. For the group homomorphism κ of the previous exercise describe the induced map κ_*.