Module 4 : Theory Covering Spaces

Lecture 15 : Covering Projections

Exercises:

1. Explain why the map $\phi : \mathbb{C} - \{0, 1/2\} \longrightarrow \mathbb{C} - \{-1/4\}$ given by $\phi(z) = z(z-1)$ is not a covering projection?
2. Show that the map $f : S^1 \longrightarrow S^1$ given by $f(z) = z^k$ is a covering projection for every $k \in \mathbb{N}$.
3. Suppose $p : \tilde{X} \longrightarrow X$ is a covering projection and E is a closed subset of X. Is the map
$$p : \tilde{X} - p^{-1}(E) \longrightarrow X - E$$
a covering projection?
4. Find a discrete subset E of $\mathbb{C}$ such that $\sin : \mathbb{C} - E \longrightarrow \mathbb{C} - \{-1, 1\}$ is a covering projection.
5. Suppose that $p : \tilde{X} \longrightarrow X$ and $q : \tilde{Y} \longrightarrow Y$ are covering projections then the product map $(p, q) : \tilde{X} \times \tilde{Y} \longrightarrow X \times Y$ given by
$$(p, q)(z, w) = (p(z), q(w)), \quad z \in \tilde{X}, w \in \tilde{Y},$$
is a covering projection. In particular the plane $\mathbb{R}^2$ is a covering space of the torus $S^1 \times S^1$.
6. Let Y be the infinite grid
$$Y = \{(x, y) \in \mathbb{R}^2 / x \in \mathbb{Z} \quad \text{or } y \in \mathbb{Z}\}$$
is a covering projection of the figure eight loop. Draw the figure eight loop on the torus.
7. Show that the set G in theorem (15.2) is closed without using the Hausdorff assumption on T.