Module 3 : Fundamnetal groups and it's basic properties

Lecture 12 & 13 :The fundamental group of the circle.

Exercises:

1. Formulate and prove the Borsuk Ulam theorem for continuous maps from S^1 to the real line.

2. Use the Borsuk Ulam theorem to prove that a pair of homogeneous polynomials of odd degree in three real variables have a common non-trivial zero.

3. For the following three maps $f : S^1 \longrightarrow S^1$ compute the induced map

   $f_* : \pi_1(S^1, 1) \longrightarrow \pi_1(S^1, 1)$. All three maps preserve the base point 1.

   (i) $f(z) = z^n$

   (ii) $f(z) = \bar{z}$.

   (iii) $f(z) = \frac{z^2 - z + \frac{3}{2}}{|z^2 - z + \frac{3}{2}|}$. Hint: Is $(z^2 - z)t + 3/2 = 0$ for any $z \in S^1$ and

   $0 \leq t \leq 1$?

4. Let X be the union of the sphere S^2 and one of its diameters. Use exercise 1 of lecture 8 to determine a generator for $\pi_1(X, x_0)$, where x_0 is a point on the sphere.

5. Determine the generators of the group $\pi_1(S^1 \times S^1, (1, 1))$. Determine the generators

   for the fundamental group of the space X of example 11.3.

6. Compute $f_* : \pi_1(\mathbb{C} - \{0\}, 1) \longrightarrow \pi_1(\mathbb{C} - \{0\}, 1)$ for the function $f(z) = z^k$.