Module 3 : Fundamental groups & its basic properties

Lecture 10 : Brouwer's theorem and its applications

Exercises:

- 1. Suppose that a space X has the fixed point property, is it necessary that it be connected? Does it have to be path-connected?
- 2. Explain why a non-trivial topological group cannot have the fixed point property.
- 3. Prove the Brouwer's fixed point theorem for the closed unit ball in \mathbb{R}^n given that that there exists a functor T from the category **Top** to the category **AbGr** such that

T(X) is the trivial group for every convex subset X of a Euclidean space and

 $T(S^{n-1})$ is a non-trivial group.

- 4. Show that the Brouwer's fixed point theorem implies the no retraction theorem.
- 5. Explain how the homotopies F_j in the proof of theorem 10.4 can be juxtaposed.

6. Show that the circle S^1 is not a retract of the sphere S^2 .