Problem Set 2 : Ruler and Compass Constructions

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(1) The construction of a regular 7-gon amounts to the construction of the real number $\alpha = \cos(2\pi/7)$. Show that α is a root of $f(x) = x^3 + x^2 - 2x - 1$. Hence conclude that a regular 7-gon is not constructible by ruler and compass.

(2) Show that $\alpha = 2\cos(2\pi/5)$ satisfies the equation $x^2 + x - 1 = 0$. Conclude that a regular 5-gon is constructible by ruler and compass. Describe a ruler and compass construction of the regular pentagon.

(3) Show that it is impossible to construct a regular 9-gon by ruler and compass by first principles.

(4) Show that an angle of n degrees, $n \in \mathbb{N}$, is constructible if and only if $3 \mid n$.

(5) Prove that it is impossible, in general, to quintsect an arbitrary angle by ruler an compass. Is it possible to divide the angle 60 degrees into five equal parts by ruler and compass ?

(6) Without using Gauss's theorem show that if m and n are coprime natural numbers and regular m-gon and n-gon are constructible, then we can construct a regular mn-gon by ruler and compass.

(7) Let p/q be a rational number written in lowest terms. Show that for an angle of p/q degrees to be constructible, it is necessary and sufficient that 3 divides p and $q = 2^k p_1 p_2 \dots p_t$ where $p_1, p_2, \dots, p_t$ are distinct Fermat primes other than 3 or 5.