Module 4

Short Circuit analysis

4.1 $\bar{\mathbf{Z}}_{\text{BUS}}$ formation without mutual coupling between elements

For a network with 'm' buses and a reference bus, one can write a relation between bus currents and bus voltages as

$$\left[\bar{\mathbf{I}}_{\mathbf{BUS}}\right] = \left[\bar{\mathbf{Y}}_{\mathbf{BUS}}\right]\left[\bar{\mathbf{V}}_{\mathbf{BUS}}\right] \tag{4.1}$$

Where,

$\bar{\mathbf{I}}_{\mathbf{BUS}}$ is $(m \times 1)$ bus current injection vector

$\bar{\mathbf{V}}_{\mathbf{BUS}}$ is $(m \times 1)$ bus voltage vector

$\bar{\mathbf{Y}}_{\mathbf{BUS}}$ is $(m \times m)$ bus admittence matrix

equation (4.1) can also be written as

$$\left[\bar{\mathbf{V}}_{\mathbf{BUS}}\right] = \left[\bar{\mathbf{Z}}_{\mathbf{BUS}}\right]\left[\bar{\mathbf{I}}_{\mathbf{BUS}}\right] \tag{4.2}$$

Where,

$\bar{\mathbf{Z}}_{\mathbf{BUS}}$ is $m \times m$ bus impedance matrix and is given by,

$$\left[\bar{\mathbf{Z}}_{\mathbf{BUS}}\right] = \left[\bar{\mathbf{Y}}_{\mathbf{BUS}}\right]^{-1}$$

From equation (4.2) for the i^{th} bus one can write

$$\bar{V}_i = \bar{Z}_{i1}\bar{I}_1 + \bar{Z}_{i2}\bar{I}_2 + \cdots \bar{Z}_{ii}\bar{I}_i + \cdots + \bar{Z}_{im}\bar{I}_m \tag{4.3}$$

From equation (4.3), $\bar{Z}_{ij}$ can be written as

$$\bar{Z}_{ij} = \left.\frac{\bar{V}_i}{\bar{I}_j}\right|_{\bar{I}_k = 0;\ \forall\ k = 1, 2, \cdots m,\ \neq j} \tag{4.4}$$

$$\bar{Z}_{ii} = \frac{\bar{V}_i}{\bar{I}_i}\Bigg|_{\bar{I}_k = 0;\ \forall\ k = 1, 2, \cdots m,\ \neq i} \tag{4.5}$$

Following points should be noted for the $\bar{\mathbf{Z}}_{\mathbf{BUS}}$ matrix

- $\bar{Z}_{ij}$ is the off-diagonal element of $\bar{\mathbf{Z}}_{\mathbf{BUS}}$ matrix and is called the 'open-circuit transfer impedance' between i^{th} and j^{th} bus.

- $\bar{Z}_{ii}$ is the diagonal element of $\bar{\mathbf{Z}}_{\mathbf{BUS}}$ matrix and is called the 'open-circuit driving point impedance' of i^{th} bus.

- If the $\bar{\mathbf{Y}}_{\mathbf{BUS}}$ matrix is symmetrical, then the matrix $\bar{\mathbf{Z}}_{\mathbf{BUS}}$ is also symmetrical i.e. $\bar{Z}_{ik} = \bar{Z}_{ki}$.

- Since in a power network each bus is connected to very few other buses,the $\bar{\mathbf{Y}}_{\mathbf{BUS}}$ matrix of the network has large number of zero elements and is therefore, sparse in nature. The $\mathbf{Z}_{\mathbf{BUS}}$, matrix on the other hand, is invariably a full matrix.

The $\bar{\mathbf{Z}}_{\mathbf{BUS}}$ matrix of a network can be found out by inverting the $\bar{\mathbf{Y}}_{\mathbf{BUS}}$ matrix of the network. This is not an efficient method as every time there is a modification in the network, the $\bar{\mathbf{Y}}_{\mathbf{BUS}}$ matrix is modified and inversion has to be done again to obtain the modified the $\bar{\mathbf{Z}}_{\mathbf{BUS}}$ matrix.

A step-by-step $\bar{\mathbf{Z}}_{\mathbf{BUS}}$ building algorithm overcomes these problems. It avoids the inversion process and network modifications are easily incorporated in the existing $\bar{\mathbf{Z}}_{\mathbf{BUS}}$.

Few terms need to be defined before the step by step process can be explained. These are :

- **Graph** : The graph of a network describes the geometrical structure of the network showing the interconnections of network elements.

- **Tree** : A tree of a graph is a connected sub graph that connects all the nodes without forming a closed path or a loop. A graph can have a number of distinct trees.

- **Branches** : The elements of a tree are called branches. The number of branches 'b' of a tree with 'n' nodes, including reference, is given by

$$b = n - 1 \tag{4.6}$$

- **Links** : The elements of a graph not included in the tree of the graph are called links. Each link is associated with a loop. If 'e' is the number of elements in a graph, then the number of links 'ℓ' is given by

$$\ell \ = \ e - b \ = \ e - n + 1 \tag{4.7}$$

The above definitions are explained with the help of illustrations as shown below :

Fig. 4.1 is a single line diagram of a power system. It has 4 buses, bus(1) to bus(4) and six elements *element e_1 to element e_6* . In this figure, bus(0) is taken as the reference bus.

Fig. 4.2 shows the graph of the network depicting the interconnection of the elements and the reference node.

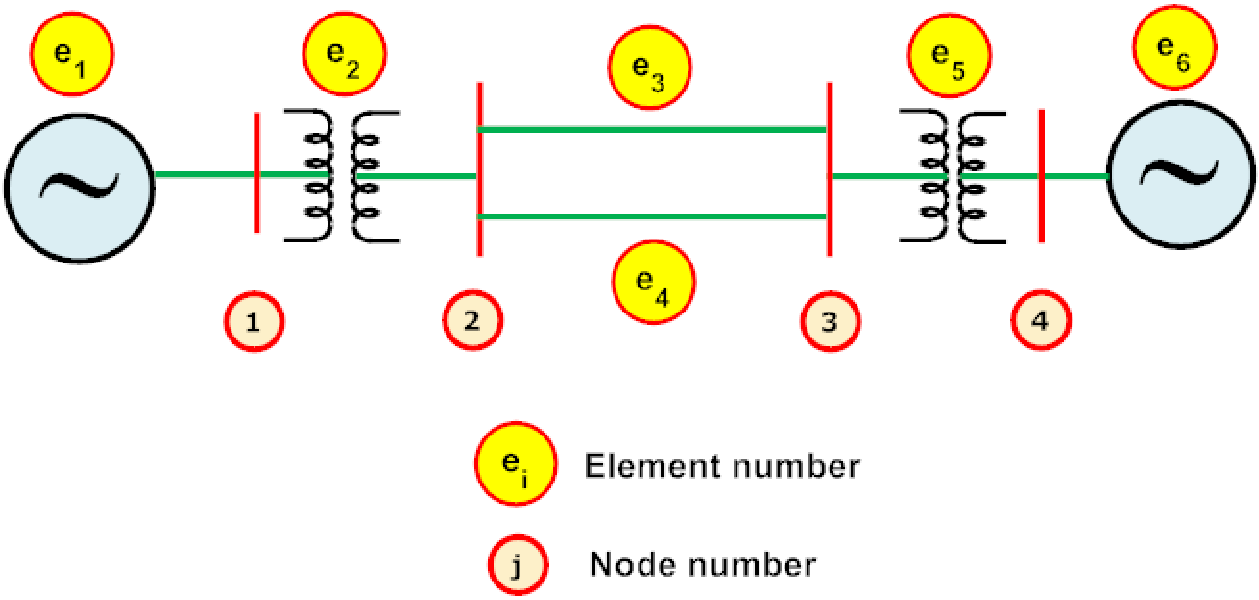

Figure 4.1: Single Line Diagram of a Power System

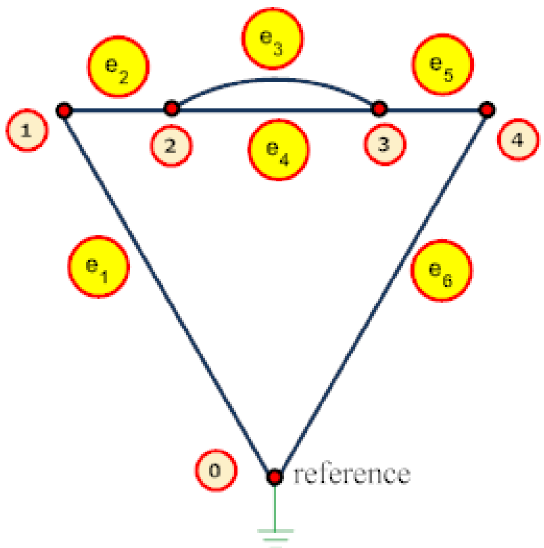

Figure 4.2: A graph of the Power system of Fig. 4.1

A tree of the graph of Fig. 4.2 is shown in Fig. 4.3. The branches and the links have been shown with solid lines and dotted lines respectively.

Following points should be noted from Fig. 4.3 :

- The total number of nodes (including reference node) is 5 (i.e. $n = 5$)
- The number of branches is $b = n - 1 = 5 - 1 = 4$. As can be as in Fig. 4.3 where

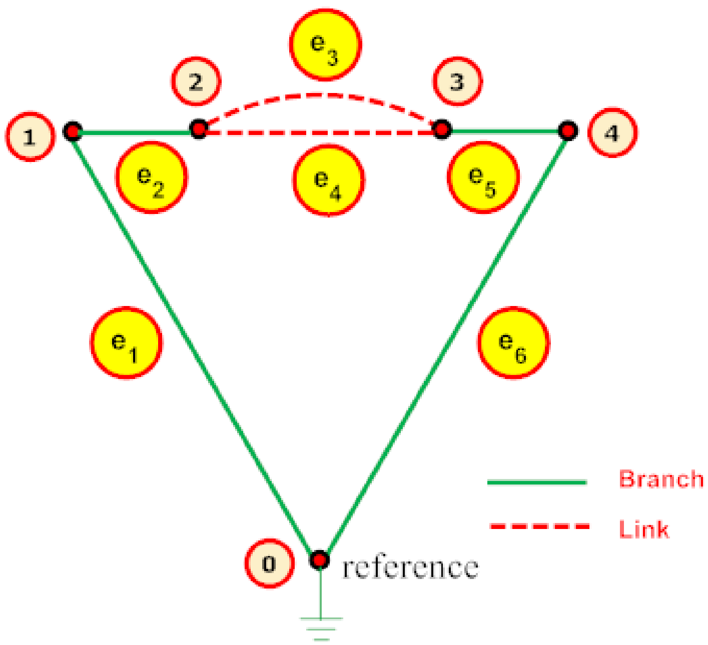

Figure 4.3: A tree of the graph of Fig. 4.2

$e_1,\ e_2,\ e_5,\ e_6,$ are such a set of branches that form a tree of the graph.

- The total number of elements in the graph is $e = 6$.
- The number of links is $\ell = e - n + 1 = 6 - 5 + 1 = 2$. The two links in the graph are e_3 and e_4 shown with dotted lines in Fig. 4.3.

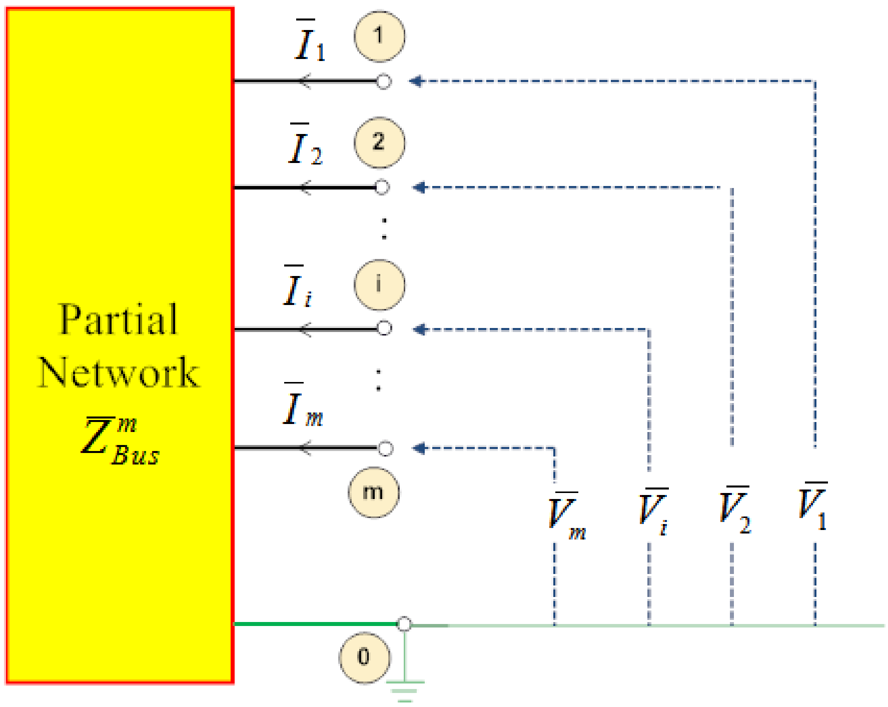

Figure 4.4: Partial network with 'm' buses

The bus impedance matrix is built up starting with a branch connected to the reference and subsequently the elements are added one by one till all the nodes and elements are considered. Let us assume that the $\bar{\mathbf{Z}}_{\mathbf{BUS}}$ matrix for a partial network with 'm' buses and a reference bus '0', as shown in Fig. 4.4, exists.

The bus voltages and bus currents for the partial network satisfy the relation

$$\left[\bar{\mathbf{V}}^{\mathbf{m}}_{\mathbf{BUS}}\right] = \left[\bar{\mathbf{Z}}^{\mathbf{m}}_{\mathbf{BUS}}\right]\left[\bar{\mathbf{I}}^{\mathbf{m}}_{\mathbf{BUS}}\right] \tag{4.8}$$

Where,

$\bar{\mathbf{V}}^{\mathbf{m}}_{\mathbf{BUS}}$ is $m \times 1$ bus voltage vector

$\bar{\mathbf{I}}^{\mathbf{m}}_{\mathbf{BUS}}$ is $m \times 1$ bus current injection vector

$\bar{\mathbf{Z}}^{\mathbf{m}}_{\mathbf{BUS}}$ is $m \times m$ bus impedence matrix of the partial network

To build $\bar{\mathbf{Z}}_{\mathbf{BUS}}$, one element at a time is added to the partial network, till all the elements are added to the network. The added element may be a branch or a link and hence the four possible element additions to a partial network are:

a. Addition of a branch between a new node and the reference

b. Addition of a branch between a new node and an existing node

c. Addition of a link between an existing node and the reference

d. Addition of a link between two existing nodes

Let us now discuss these four cases one-by-one in detail.

4.1.1 Addition of a branch between a new node and the reference node (case 1):

Fig. 4.5 shows the addition of a branch between a new node 'q' and the reference '0'.The addition of a new node to the partial network increases the size of $\bar{\mathbf{Z}}_{\mathbf{BUS}}$ to $(m+1)\times(m+1)$ with the addition of a new row and a new column corresponding to the new node 'q', Let the impedance of this branch be $\bar{z}_{q0}$. The new network equation can be written as:

$$\begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \vdots \\ \bar{V}_p \\ \vdots \\ \bar{V}_m \\ \cdots \\ \bar{V}_q \end{bmatrix} = \begin{bmatrix} \bar{Z}_{11} & \bar{Z}_{12} & \cdots & \bar{Z}_{1p} & \cdots & \bar{Z}_{1m} & \vdots & \bar{Z}_{1q} \\ \bar{Z}_{21} & \bar{Z}_{22} & \cdots & \bar{Z}_{2p} & \cdots & \bar{Z}_{2m} & \vdots & \bar{Z}_{2q} \\ \vdots & & & & & & \vdots & \\ \bar{Z}_{p1} & \bar{Z}_{p2} & \cdots & \bar{Z}_{pp} & \cdots & \bar{Z}_{pm} & \vdots & \bar{Z}_{pq} \\ \vdots & & & & & & & \\ \bar{Z}_{m1} & \bar{Z}_{m2} & \cdots & \bar{Z}_{mp} & \cdots & \bar{Z}_{mm} & \vdots & \bar{Z}_{mq} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \bar{Z}_{q1} & \bar{Z}_{q2} & \cdots & \bar{Z}_{qp} & \cdots & \bar{Z}_{qm} & \vdots & \bar{Z}_{qq} \end{bmatrix} \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \vdots \\ \bar{I}_p \\ \vdots \\ \bar{I}_m \\ \cdots \\ \bar{I}_q \end{bmatrix} \tag{4.9}$$

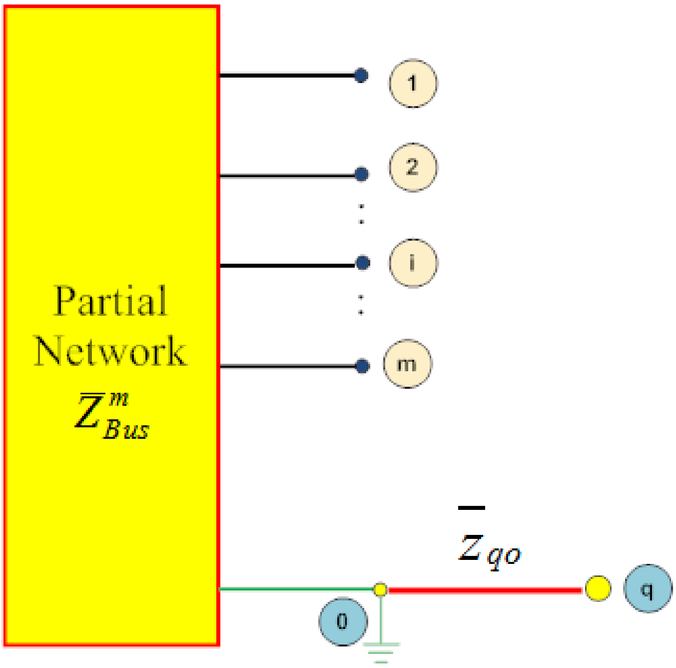

Figure 4.5: Addition of a branch between a new node and the reference

The addition of branch does not change the elements of the original matrix $\bar{\mathbf{Z}}_{\mathbf{BUS}}$. Only the elements of the added new row and column corresponding to q^{th} bus need to be calculated. Further,since the power system elements are linear and bilateral, $\bar{Z}_{qi} = \bar{Z}_{iq}, \ \forall \ i = 1, \ 2, \ \cdots m.$

Now since,

$$\bar{Z}_{qq} = \left. \frac{\bar{V}_q}{\bar{I}_q} \right|_{\bar{I}_k = 0; \ \forall \ k = 1, 2, \cdots m}$$

a current source of $\bar{I}_q = 1$ p.u is connected to the q^{th} bus, with all the others buses open, and the voltage of q^{th} bus $(\bar{V}_q)$ is computed, as shown in Fig. 4.6.

From Fig. 4.6 one gets $\bar{V}_q = \bar{z}_{q0}\bar{I}_q$, and thus with $\bar{I}_q = 1 \ p.u.$

$$\bar{Z}_{qq} = \left. \frac{\bar{V}_q}{\bar{I}_q} \right|_{\bar{I}_k = 0; \ \forall \ k = 1, 2, \cdots m,} = \bar{z}_{qo}$$

For finding out $\bar{Z}_{qi}$, a current source $\bar{I}_i = 1$ p.u. is connected between i^{th} bus and the reference bus with all other buses open circuited as shown in Fig. 4.7.

From Fig. 4.7 , $\bar{V}_q = 0$, and hence with $\bar{I}_i = 1 \ p.u.$

$$\bar{Z}_{qi} = \left. \frac{\bar{V}_q}{\bar{I}_i} \right|_{\bar{I}_k = 0; \ \forall \ k = 1, 2, \cdots m, \ \neq i} = 0$$

This implies that all the off-diagonal elements $\bar{Z}_{q1}, \ \bar{Z}_{q2}, \ \cdots \ \bar{Z}_{mq}$ and $\bar{Z}_{1q}, \ \bar{Z}_{2q}, \ \cdots \ \bar{Z}_{qm}$ are equal to zero.

Hence, the modified $\bar{\mathbf{Z}}_{\mathbf{Bus}}$ matrix after addition of an element between the new bus 'q' and the

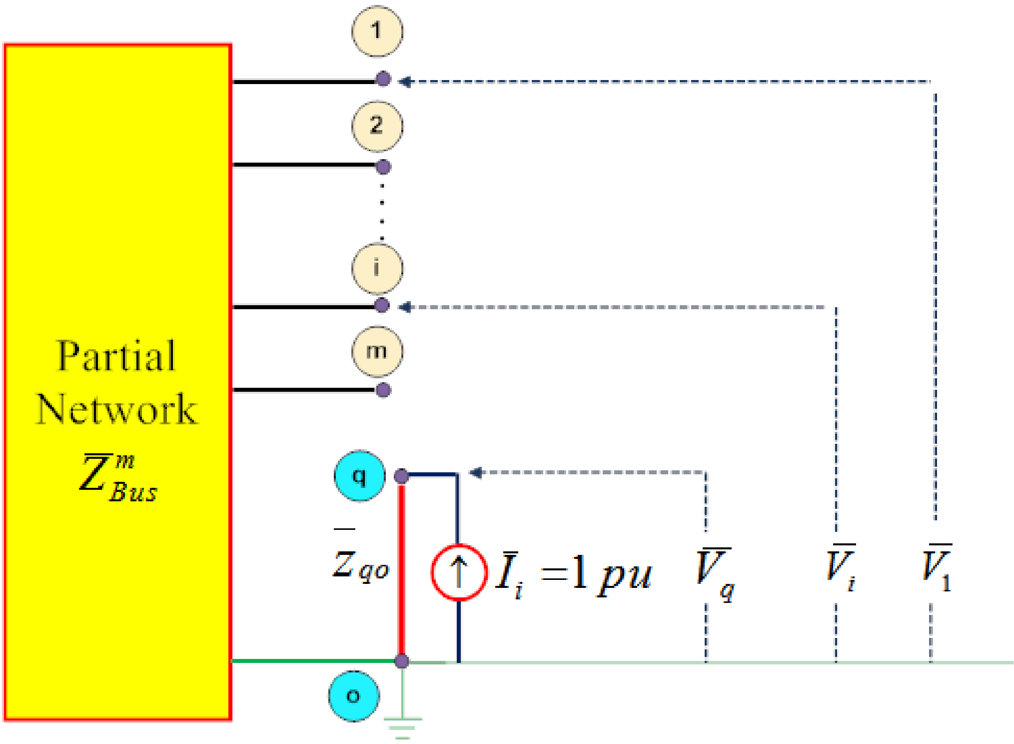

Figure 4.6: Calculation of $\bar{Z}_{qq}$ for Case 1

reference bus '0′ is given as,

$$\bar{\mathbf{Z}}_{\mathbf{BUS}} = \begin{bmatrix} \bar{Z}_{11} & \bar{Z}_{12} & \cdots & \bar{Z}_{1p} & \cdots & \bar{Z}_{1m} & 0 \\ \bar{Z}_{21} & \bar{Z}_{22} & \cdots & \bar{Z}_{2p} & \cdots & \bar{Z}_{2m} & 0 \\ \vdots & & & & & & \\ \bar{Z}_{p1} & \bar{Z}_{p2} & \cdots & \bar{Z}_{pp} & \cdots & \bar{Z}_{pm} & 0 \\ \vdots & & & & & & \\ \bar{Z}_{m1} & \bar{Z}_{m2} & \cdots & \bar{Z}_{mp} & \cdots & \bar{Z}_{mm} & 0 \\ 0 & 0 & \cdots & 0 & \cdots & 0 & \bar{z}_{qo} \end{bmatrix} \tag{4.10}$$

4.1.2 Addition of a branch between a new node and an existing node (Case 2):

Let a branch with impedance $\bar{z}_{pq}$ be connected between an existing node 'p' and a new node 'q' as shown in Fig. 4.8. In this case also, the size of $\bar{\mathbf{Z}}_{\mathbf{Bus}}$ matrix increases by one to $(m+1)\times(m+1)$ due to the addition of a new node 'q' to the network. The modified network equations can be written

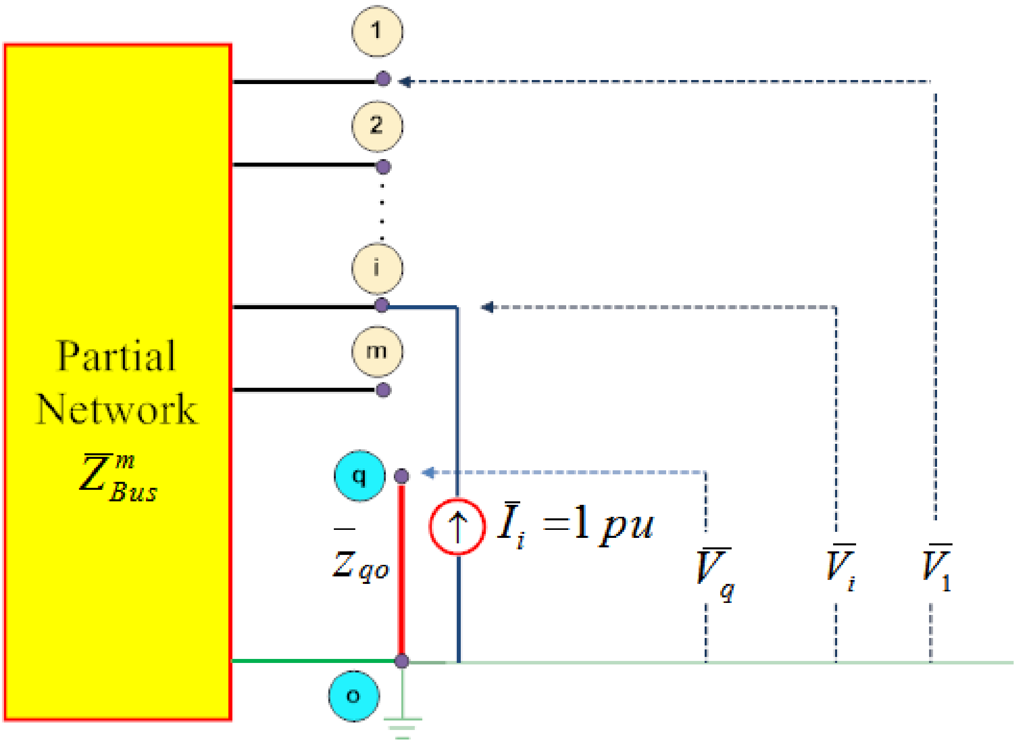

Figure 4.7: Calculation of $\bar{Z}_{qi}$ for case 1

as:

$$\begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \vdots \\ \bar{V}_p \\ \vdots \\ \bar{V}_m \\ \cdots \\ \bar{V}_q \end{bmatrix} = \begin{bmatrix} \bar{Z}_{11} & \bar{Z}_{12} & \cdots & \bar{Z}_{1p} & \cdots & \bar{Z}_{1m} & \vdots & \bar{Z}_{1q} \\ \bar{Z}_{21} & \bar{Z}_{22} & \cdots & \bar{Z}_{2p} & \cdots & \bar{Z}_{2m} & \vdots & \bar{Z}_{2q} \\ \vdots & & & & & & \vdots & \\ \bar{Z}_{p1} & \bar{Z}_{p2} & \cdots & \bar{Z}_{pp} & \cdots & \bar{Z}_{pm} & \vdots & \bar{Z}_{pq} \\ \vdots & & & & & & \vdots & \\ \bar{Z}_{m1} & \bar{Z}_{m2} & \cdots & \bar{Z}_{mp} & \cdots & \bar{Z}_{mm} & \vdots & \bar{Z}_{mq} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \bar{Z}_{q1} & \bar{Z}_{q2} & \cdots & \bar{Z}_{qp} & \cdots & \bar{Z}_{qm} & \vdots & \bar{Z}_{qq} \end{bmatrix} \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \vdots \\ \bar{I}_p \\ \vdots \\ \bar{I}_m \\ \cdots \\ \bar{I}_q \end{bmatrix} \tag{4.11}$$

Even after the addition of branch p-q, the original matrix $\mathbf{\bar{Z}^m_{Bus}}$ remains unchanged. Only the additional elements corresponding to the q^{th} row and column need to be calculated.

For calculating $\bar{Z}_{qq}$ one can write

$$\bar{Z}_{qq} = \left.\frac{\bar{V}_q}{\bar{I}_q}\right|_{\bar{I}_k = 0;\ \forall\ k = 1,2,\cdots m}$$

To evaluate $\bar{Z}_{qq}$ current source of $\bar{I}_q = 1$ p.u is connected to the q^{th} bus, with all the others buses open circuited, and the voltage of q^{th} bus $\bar{V}_q$ is computed, as shown in Fig. 4.9. From Fig. 4.9 with

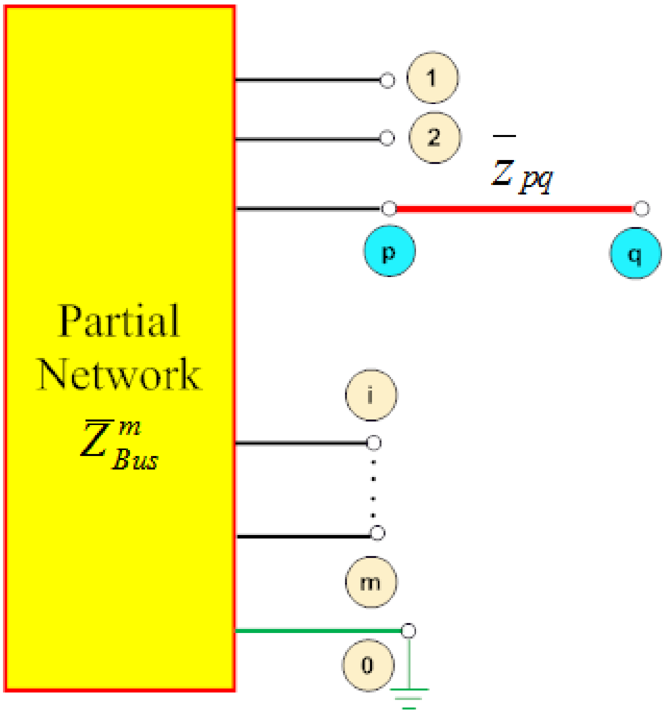

Figure 4.8: Addition of a branch between an existing node 'p' and a new node 'q'

$\bar{I}_q = 1\ p.u.$ and $\bar{I}_k = 0,\ \forall\ k = 1,\ 2,\ \cdots,\ m$ one can write,

$$\left.\begin{array}{c} \bar{V}_1 = \bar{Z}_{1q}\bar{I}_q = \bar{Z}_{1q} \\ \bar{V}_2 = \bar{Z}_{2q}\bar{I}_q = \bar{Z}_{2q} \\ \vdots \\ \bar{V}_p = \bar{Z}_{pq}\bar{I}_q = \bar{Z}_{pq} \\ \vdots \\ \bar{V}_m = \bar{Z}_{mq}\bar{I}_q = \bar{Z}_{mq} \\ \bar{V}_q = \bar{Z}_{qq}\bar{I}_q = \bar{Z}_{qq} \end{array}\right\} \tag{4.12}$$

From the Fig. 4.10, the voltages $\bar{V}_p$ and $\bar{V}_q$ can be related as

$$\bar{V}_q = \bar{V}_p - \bar{v}_{pq} = \bar{Z}_{pq}\bar{I}_q - \bar{z}_{pq}\bar{i}_{pq} = \bar{Z}_{pq} + \bar{z}_{pq} \tag{4.13}$$

Because, from the Fig. 4.10 , $\bar{i}_{pq} = -\bar{I}_q = -1\ pu$ and from equation (4.12) $\bar{V}_p = \bar{Z}_{pq}$ and $\bar{V}_q = \bar{Z}_{qq}$. Thus,

$$\boxed{\bar{Z}_{qq} = \bar{Z}_{pq} + \bar{z}_{pq}} \tag{4.14}$$

For calculating $\bar{Z}_{qi}$ one can write

$$\bar{Z}_{qi} = \left.\frac{\bar{V}_q}{\bar{I}_i}\right|_{\bar{I}_k = 0;\ \forall\ k = 1,\,2,\,\cdots\,m,\ \neq i}$$

Hence, to compute $\bar{Z}_{qi}$ a current source of $\bar{I}_1 = 1$ p.u is connected to the i^{th} bus, with all the others

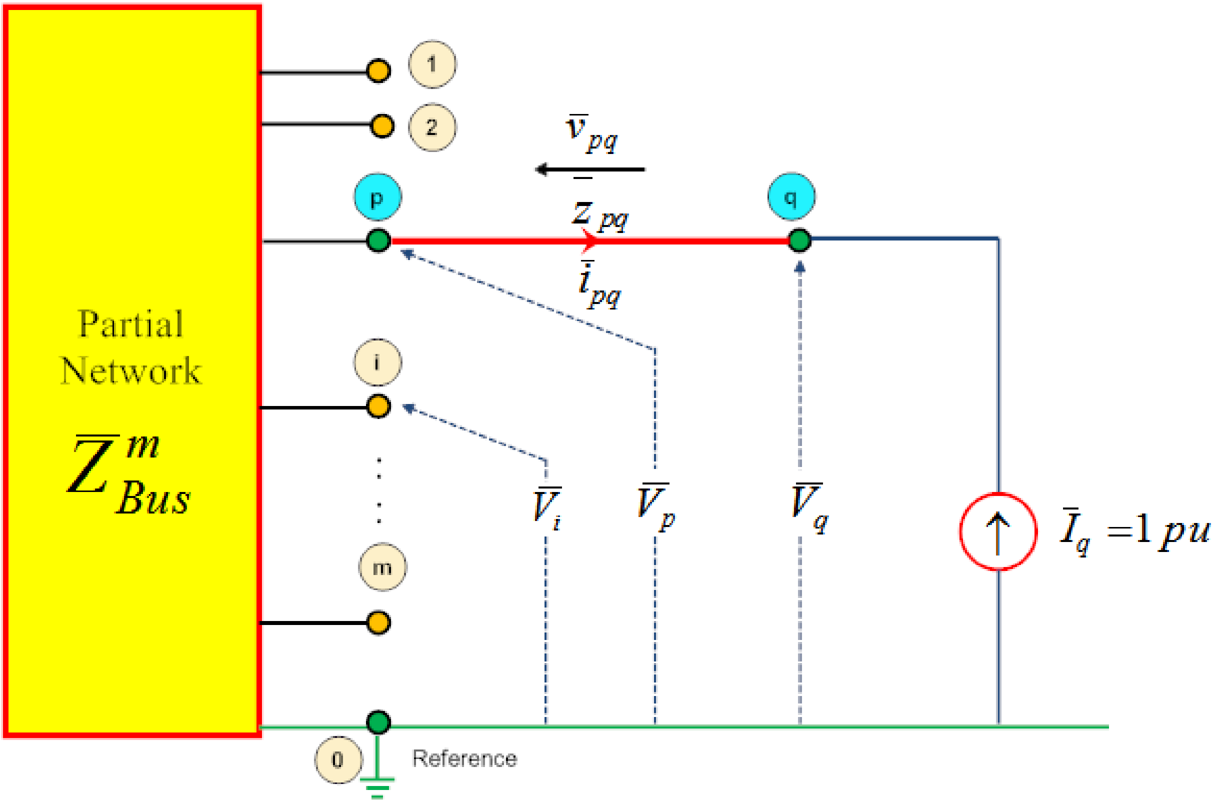

Figure 4.9: Calculation of $\bar{Z}_{qq}$

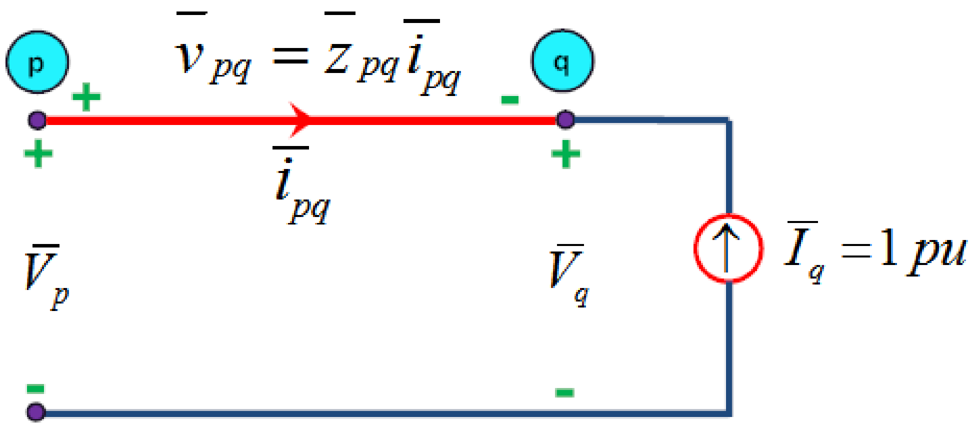

Figure 4.10: Relation between $\bar{V}_p$ and $\bar{V}_q$

buses open circuited, and the bus voltage $\bar{V}_i$ is computed for all the buses, as shown in Fig. 4.11. From equation (4.11) one gets

$$\left.\begin{array}{c} \bar{V}_1 = \bar{Z}_{1i}\bar{I}_i = \bar{Z}_{1i} \\ \bar{V}_2 = \bar{Z}_{2i}\bar{I}_i = \bar{Z}_{2i} \\ \vdots \\ \bar{V}_p = \bar{Z}_{pi}\bar{I}_i = \bar{Z}_{pi} \\ \vdots \\ \bar{V}_m = \bar{Z}_{mi}I_i = \bar{Z}_{mi} \\ \bar{V}_q = \bar{Z}_{qi}\bar{I}_i = \bar{Z}_{qi} \end{array}\right\}$$

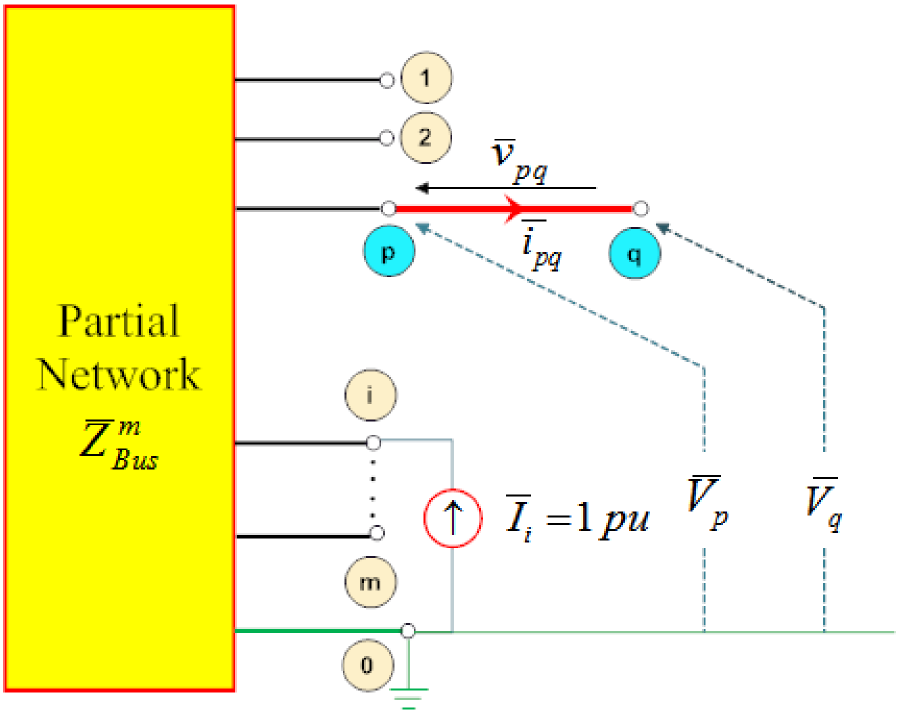

Figure 4.11: Calculation of $\bar{Z}_{qi}$ for case 2

From Fig. 4.11, $\bar{V}_q = \bar{V}_p$ as the current in the branch $p - q$ is zero. Hence, from the above equations one gets

$$\boxed{\bar{Z}_{qi} = \bar{Z}_{pi};\ \forall\ i\ =\ 1,\ 2,\ \cdots m} \tag{4.15}$$

Hence, the modified $\mathbf{\bar{Z}_{Bus}}$ matrix after addition of an element between an existing bus 'p' the new bus 'q' is given as,

$$\mathbf{\bar{Z}_{BUS}} = \begin{bmatrix} \bar{Z}_{11} & \bar{Z}_{12} & \cdots & \bar{Z}_{1p} & \cdots & \bar{Z}_{1m} & \bar{Z}_{1p} \\ \bar{Z}_{21} & \bar{Z}_{22} & \cdots & \bar{Z}_{2p} & \cdots & \bar{Z}_{2m} & \bar{Z}_{2p} \\ \vdots & & & & & & \\ \bar{Z}_{p1} & \bar{Z}_{p2} & \cdots & \bar{Z}_{pp} & \cdots & \bar{Z}_{pm} & \bar{Z}_{pp} \\ \vdots & & & & & & \\ \bar{Z}_{m1} & \bar{Z}_{m2} & \cdots & \bar{Z}_{mp} & \cdots & \bar{Z}_{mm} & \bar{Z}_{mp} \\ \bar{Z}_{p1} & \bar{Z}_{p2} & \cdots & \bar{Z}_{pp} & \cdots & \bar{Z}_{pm} & \bar{Z}_{pq} + \bar{z}_{qp} \end{bmatrix} \tag{4.16}$$

So far in this lecture, we have considered the cases of addition of branches only. In the next lecture we will consider the case of addition of links.