2.11 Fast-decoupled load-flow (FDLF) technique

An important and useful property of power system is that the change in real power is primarily governed by the charges in the voltage angles, but not in voltage magnitudes. On the other hand, the charges in the reactive power are primarily influenced by the charges in voltage magnitudes, but not in the voltage angles. To see this, let us note the following facts:

(a) Under normal steady state operation, the voltage magnitudes are all nearly equal to 1.0.

(b) As the transmission lines are mostly reactive, the conductances are quite small as compared to the susceptance ($G_{ij} << B_{ij}$).

(c) Under normal steady state operation the angular differences among the bus voltages are quite small ($\theta_i - \theta_j \approx 0$ (within $5^o - 10^o$)).

(d) The injected reactive power at any bus is always much less than the reactive power consumed by the elements connected to this bus when these elements are shorted to the ground ($Q_i << B_{ii}V_i^2$).

With these facts at hand, let us re-visit the equations for Jacobian elements in Newton-Raphson (polar) method (equation (2.48) to (2.55)). From equations (2.50) and (2.51) we have,

$$\begin{aligned}
\frac{\partial P_i}{\partial V_j} &= 2V_iG_{ii} + \sum_{\substack{k=1\\ \neq i}}^{n} V_kY_{ik}\cos(\theta_i - \theta_k - \alpha_{ik}) \\
&= 2V_iG_{ii} + \sum_{\substack{k=1\\ \neq i}}^{n} V_kY_{ik}\left[\cos(\theta_i - \theta_k)\cos\alpha_{ik} + \sin(\theta_i - \theta_k)\sin\alpha_{ik}\right] \\
&= 2V_iG_{ii} + \sum_{\substack{k=1\\ \neq i}}^{n} V_k\left[G_{ik}\cos(\theta_i - \theta_k) + B_{ik}\sin(\theta_i - \theta_k)\right]; \quad j = i \qquad (2.74)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial P_i}{\partial V_j} &= V_iY_{ij}\cos(\theta_i - \theta_j - \alpha_{ij}) \\
&= V_iY_{ij}\left[\cos(\theta_i - \theta_j)\cos\alpha_{ij} + \sin(\theta_i - \theta_j)\sin\alpha_{ij}\right] \\
&= V_i\left[G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)\right]; \quad j \neq i \qquad (2.75)
\end{aligned}$$

Now, G_{ii} and G_{ij} are quite small and negligible and also $\cos(\theta_i - \theta_j) \approx 1$ and $\sin(\theta_i - \theta_j) \approx 0$, as $[(\theta_i - \theta_j) \approx 0]$. Hence,

$$\frac{\partial P_i}{\partial V_i} \approx 0 \quad \text{and} \quad \frac{\partial P_i}{\partial V_j} \approx 0 \quad \Longrightarrow \mathbf{J_2} \approx 0 \qquad (2.76)$$

Similarly, from equations (2.52) and (2.53) we get,

$$\frac{\partial Q_i}{\partial \theta_j} = \sum_{\substack{k=1\\ \neq i}}^{n} V_iV_k\left[G_{ik}\cos(\theta_i - \theta_k) + B_{ik}\sin(\theta_i - \theta_k)\right]; \quad j = i \qquad (2.77)$$

$$\frac{\partial Q_i}{\partial \theta_j} = -V_iV_j\left[G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)\right]; \quad j \neq i \qquad (2.78)$$

Again in light of the natures of the quantities G_{ii}, G_{ij} and $(\theta_i - \theta_j)$ as discussed above,

$$\frac{\partial Q_i}{\partial \theta_i} \approx 0 \quad \text{and} \quad \frac{\partial Q_i}{\partial \theta_j} \approx 0 \quad \Longrightarrow \mathbf{J_3} \approx 0 \tag{2.79}$$

Substituting equations (2.76) and (2.79) into equation (2.40) one can get,

$$\boxed{\begin{bmatrix} \mathbf{\Delta P} \\ \mathbf{\Delta Q} \end{bmatrix} = \begin{bmatrix} \mathbf{J_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{J_4} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Delta\theta} \\ \boldsymbol{\Delta V} \end{bmatrix}} \tag{2.80}$$

In other words, $\mathbf{\Delta P}$ depends only on $\boldsymbol{\Delta\theta}$ and $\mathbf{\Delta Q}$ depends only on $\boldsymbol{\Delta V}$. Thus, there is a decoupling between '$\mathbf{\Delta P}$ - $\boldsymbol{\Delta\theta}$' and '$\mathbf{\Delta Q}$ - $\boldsymbol{\Delta V}$' relations. Now, from equations (2.48) and (2.49) we get,

$$\begin{aligned} \frac{\partial P_i}{\partial \theta_j} &= -\sum_{\substack{k=1 \\ \neq i}}^{n} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}); \quad j = i \\ &= V_i V_i Y_{ii} \sin(\theta_i - \theta_i - \alpha_{ii}) - \sum_{k=1}^{n} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}); \quad j = i \\ &= -B_{ii} V_i^2 - Q_i \approx -B_{ii} V_i^2; \quad j = i \quad [\text{as} \quad Q_i << B_{ii} V_i^2] \end{aligned} \tag{2.81}$$

$$\begin{aligned} \frac{\partial P_i}{\partial \theta_j} &= V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij}); \quad j \neq i \\ &= V_i V_j Y_{ij} [\sin(\theta_i - \theta_j) \cos \alpha_{ij} - \cos(\theta_i - \theta_j) \sin \alpha_{ij}]; \quad j \neq i \\ &= V_i V_j [G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)]; \quad j \neq i \\ &= -V_i V_j B_{ij}; \quad j \neq i \end{aligned} \tag{2.82}$$

Similarly, from equations (2.54) and (2.55) we get,

$$\begin{aligned} \frac{\partial Q_i}{\partial V_j} &= -2V_i B_{ii} + \sum_{\substack{k=1 \\ \neq i}}^{n} V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}); \quad j = i \\ \text{or,} \quad \frac{\partial Q_i}{\partial V_j} V_i &= -2V_i^2 B_{ii} + \sum_{\substack{k=1 \\ \neq i}}^{n} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}); \quad j = i \\ \text{or,} \quad \frac{\partial Q_i}{\partial V_j} V_i &= -V_i^2 B_{ii} + \sum_{k=1}^{n} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}) = Q_i - V_i^2 B_{ii}; \quad j = i \\ \text{or,} \quad \frac{\partial Q_i}{\partial V_j} V_i &= -V_i^2 B_{ii}; \quad j = i \quad [\text{as} \quad Q_i << B_{ii} V_i^2] \\ \text{or,} \quad \frac{\partial Q_i}{\partial V_j} &= -V_i B_{ii}; \quad j = i \end{aligned} \tag{2.83}$$

$$
\begin{aligned}
\frac{\partial Q_i}{\partial V_j} &= V_i Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij}); \quad j \neq i \\
&= V_i Y_{ij} \left[\sin(\theta_i - \theta_j)\cos\alpha_{ij} - \cos(\theta_i - \theta_j)\sin\alpha_{ij}\right]; \quad j \neq i \\
&= V_i \left[G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)\right]; \quad j \neq i \\
&\approx -V_i B_{ij}; \quad j \neq i
\end{aligned} \tag{2.84}
$$

Combining equations (2.80)-(2.82) we get, $\Delta P_i = -V_i \sum_{k=1}^{n} V_k B_{ik} \Delta\theta_k$. Or,

$$
\frac{\Delta P_i}{V_i} = -\sum_{k=1}^{n} V_k B_{ik} \Delta\theta_k \tag{2.85}
$$

Now, as $V_i \approx 1.0$ under normal steady state operating condition, equation (2.85) reduces to,

$$
\frac{\Delta P_i}{V_i} = -\sum_{k=1}^{n} B_{ik} \Delta\theta_k. \qquad \text{Or,} \ \frac{\boldsymbol{\Delta P}}{\mathbf{V}} = [-\mathbf{B}]\, \boldsymbol{\Delta\theta}. \qquad \text{Or,}
$$

$$
\boxed{\frac{\boldsymbol{\Delta P}}{\mathbf{V}} = \left[\boldsymbol{B}'\right] \boldsymbol{\Delta\theta}} \tag{2.86}
$$

Matrix $\boldsymbol{B}'$ is a constant matrix having a dimension of $(n-1) \times (n-1)$. Its elements are the negative of the imaginary part of the element (i, k) of the $\mathbf{Y_{BUS}}$ matrix where $i = 2, 3, \cdots\cdots n$ and $k = 2, 3, \cdots\cdots n$.

Again combining equations (2.80), (2.83) and (2.84) we get,

$$
\Delta Q_i = -V_i \sum_{k=1}^{n} B_{ik} \Delta V_k. \qquad \text{Or,} \ \frac{\Delta Q_i}{V_i} = -\sum_{k=1}^{n} B_{ik} \Delta V_k. \qquad \text{Or,}
$$

$$
\boxed{\frac{\boldsymbol{\Delta Q}}{\mathbf{V}} = \left[\boldsymbol{B}''\right] \boldsymbol{\Delta V}} \tag{2.87}
$$

Again, $[\boldsymbol{B}'']$ is also a constant matrix having a dimension of $(n-m) \times (n-m)$. Its elements are the negative of the imaginary part of the element (i, k) of the $\mathbf{Y_{BUS}}$ matrix where $i = (m+1), (m+2), \cdots\cdots n$ and $k = (m+1), (m+2), \cdots\cdots n$. As the matrixes $[\boldsymbol{B}']$ and $[\boldsymbol{B}'']$ are constant, it is not necessary to invert these matrices in each iteration. Rather, the inverse of these matrices can be stored and used in every iteration, thereby making the algorithm faster. Further simplification in the FDLF algorithm can be made by,

a. Ignoring the series resistances is calculating the elements of $[\boldsymbol{B}']$. Also, by omitting the elements of $[\boldsymbol{B}']$ that predominantly affect reactive power flows, i.e., shunt reactances and transformer off nominal in phase taps.

b. Omitting from $[\boldsymbol{B}'']$ the angle shifting effect of phase shifter, which predominantly affects real power flow.

In the next lecture, we will look at an example of FDLF method.