Nonlinear Control Design

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Nomenclature

Greek and Roman symbols

$\mathbb{R}$	Real numbers
$\mathbb{R}^+$	Set of positive real numbers
Z	Set of integers
Z^+	Set of positive integers
$\mathbb{R}^n$	Euclidean n-space
$\mathbb{S}^1$	unit circle
$\mathbb{S}^1 \times \mathbb{S}^1$	2-Torus
Q	configuration space
$B(x, \delta)$	Ball of radius δ centered at x

Course outline

This course deals with the analytical tools to analyze nonlinear systems. It covers

1. Mathematical preliminaries involving open and closed sets, compact set, dense set, Continuity of functions, Lipschitz condition, smooth functions, Vector space, norm of a vector, normed linear space, inner product space.

2. Well-posedness of ordinary differential equations, Lipschitz continuity and contraction mapping theorem.

3. An introduction to simple mechanical systems wherein the notion of degree-of-freedom, configuration space, configuration variables will be brought out. The state-space models of a few benchmark examples in nonlinear control will be derived using Euler-Lagrange formulation. The notion of equilibrium points and operating points leads to linearized models based on Jacobian linearization.

4. Second-order nonlinear systems occupy a special place in the study of nonlinear systems since they are easy to interpret geometrically in the plane. Here, the concept of a vector field, trajectories, vector field plot, phase-plane portrait and positively invariant sets are discussed. The classification of equilibrium points based on the eigenvalues of the linearized system will also be introduced and it will be seen why the analysis based on linearization fails in some cases.

5. Periodic solutions and the notion of limit cycles will lead us to the Bendixson's theorem and Poincaré-Bendixson criteria that provide sufficient conditions to rule-out and rule-in the existence of limit cycles respectively for a second-order system.

6. Stability is central to control system design and involves various notions of stability such as Lagrange stability, Lyapunov stability, asymptotic stability, global asymptotic stability, exponential stability and instability. The tools that we will use to

infer the stability properties include Lyapunov's direct and indirect method and La Salle's invariance property.

7. Two control design techniques, one based on Lyapunov function and the other on sliding mode are illustrated with examples in the final module.

Lecture-1

Introduction

All systems are inherently nonlinear in nature. This course deals with the analysis of nonlinear systems. The need for special tools to analyze nonlinear systems arises from the fact that the *principle of superposition* on which linear analysis is based, fails in the nonlinear case. This is just one reason for resorting to nonlinear analysis. Recall, the basic circuit analysis techniques such as the nodal, Thévenin, Norton etc. are applicable only for linear circuits.

The course subsumes some level of mathematical exposure to tools from set theory and calculus. The requisite mathematical preliminaries that are used in this course are brought out in the first module. Most physical systems can be modeled using ordinary differential equations. In the study of differential equations, it is natural to seek answers to questions on the existence, uniqueness etc. of solutions prior to finding a method to solve the equation. In the second module, we address these questions and seek answers to some of these them using the notion of Lipschitz continuity.

Many physical systems can be modeled accurately using the laws of Physics. With a bias towards mechanical systems, a plethora of simple mechanical systems are worked in the third Module. All the information needed to describe the motion of a system lies in the total energy, which is constituted of the kinetic and potential energies. The examples that are presented often serve as benchmark problems in control.

The study of second-order, time-invariant differential equations is given importance due to the easy visualization of the system behaviour on the plane. A major part of the study involves nature of trajectories in the vicinity of equilibrium points or the fixed points. The characterization of different equilibria and the study of the local behaviour by the method of linearization is recognized as an important and a first step in nonlinear analysis. In module 4, the focus is on second-order systems. Some nonlinear equations exhibit many interesting phenomena such as *limit cycles* and chaos. The former is the topic of study in the fifth module, which is devoted to a much general class of solutions, namely the periodic solutions.

Stability is central to control systems and has been the subject of study since the works of Alexander Lyapunov and James Clerk Maxwell. Maxwell, a Scottish physicist and mathematician, in 1868 analyzed the stability properties of the Watt's governor, 140 years after the invention of the steam engine. Much of nonlinear stability theory that is used extensively nowadays, called the second method of Lyapunov is attributed to the work by Lyapunov, a Russian mathematician who published his book "The General Problem of Stability of Motion" in 1892. The method makes use of energy-like functions that are sign-definite (positive) and looking at the sign of their gradients along the trajectories of the system. The last module is devoted the stability analysis of nonlinear systems.