

In Practice , the Diffusion of Impurities
is a Two-Step Process

(i) Predeposition (ii) Drive-In

(i) Predeposition :

Here we perform Constant Source diffusion at
Temperature T_1 for time t_1

$$\therefore D_1 = D_1(T_1) , N_{01} = N_0(T_1)$$

$$\therefore N(x, t_1) = N_{01} \operatorname{erfc} \left(\frac{x}{2\sqrt{D_1 t_1}} \right)$$

And after time t_1 , the dose of Impurities $Q(t_1)$
at Surface $= 2 N_{01} \sqrt{\frac{D_1 t_1}{\pi}}$



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(ii) After Predeposition step, source of impurities are stopped. Then the wafers are reintroduced in the furnace at Temperature T_2 for time t_2 this process is continued. This process is called Drive-In.

$$\text{At } T_2, D_2(T_2) = D_2, \therefore Dt = D_2 t_2$$

In Drive-In, no new impurities are introduced, but available ones from Predeposition are Re-distributed (Q dose). This is the case of Limited Source Diffusion giving Gaussian Profile:

$$N(x, t_1, t_2) = \frac{Q}{\sqrt{\pi D_2 t_2}} \exp\left(-\frac{x^2}{4 D_2 t_2}\right)$$

$$\propto N(x, t_1, t_2) = \frac{2 N_{01}}{\pi} \sqrt{\frac{D_1 t_1}{D_2 t_2}} \exp\left(-\frac{x^2}{4 D_2 t_2}\right)$$



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Assumption here is

$$D_1 t_1 \ll D_2 t_2$$

However even if this condition is not met, we can still evaluate ρ_{hole} using Smith's Integral.

we define $\alpha = \sqrt{\frac{D_1 t_1}{D_2 t_2}}$

and $\beta = \frac{x^2}{4(D_1 t_1 + D_2 t_2)}$

If $D_1 t_1 \ll D_2 t_2$, then

$$N(x, t_1, t_2) = \frac{2 N_{01}}{\pi} \int_{\frac{x}{\sqrt{\beta}}}^{\infty} e^{-y^2} \operatorname{erf}(\alpha y) dy$$

By simplification

$$N(x, t_1, t_2) = \frac{2 N_{01}}{\pi} \int_0^{\alpha} \frac{e^{-\beta(1+\alpha^2)}}{1+\alpha^2} d\alpha$$



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Now surface conc. (at $x=0$) N_s is given by

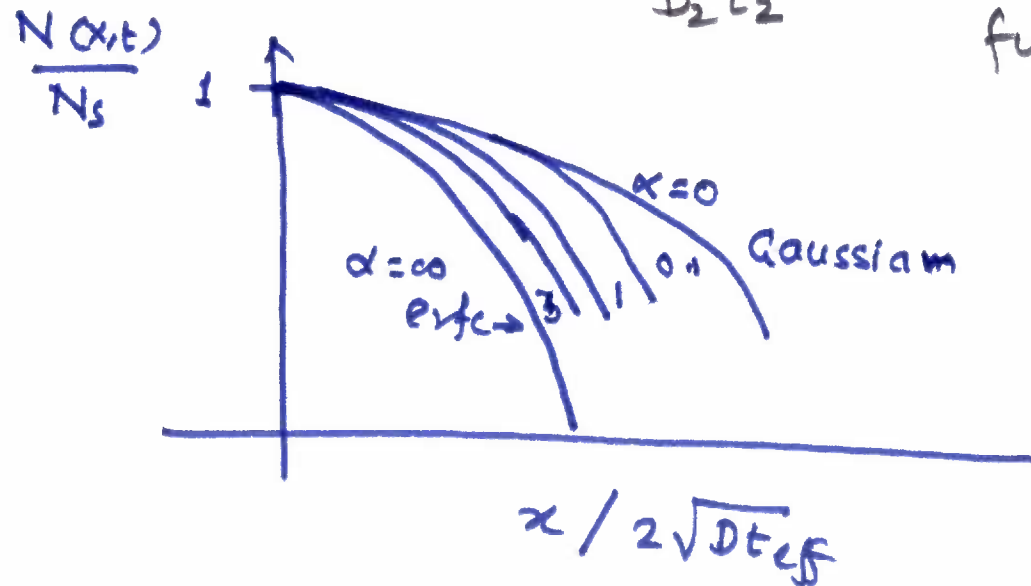
$$N_s = \frac{2N_{01}}{\pi} \tan^{-1} \alpha$$

If α is small and $\ll 1 \Rightarrow D_1 t_1 \ll D_2 t_2$

Then $\tan^{-1} \alpha = \alpha$

$$\therefore N_s = \frac{2N_{01}}{\pi} \sqrt{\frac{D_1 t_1}{D_2 t_2}}$$

Same as Standard Two step function.



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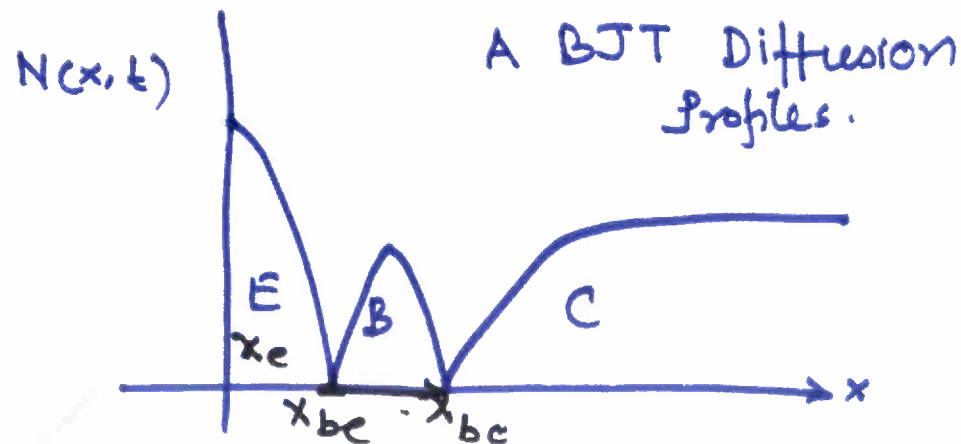
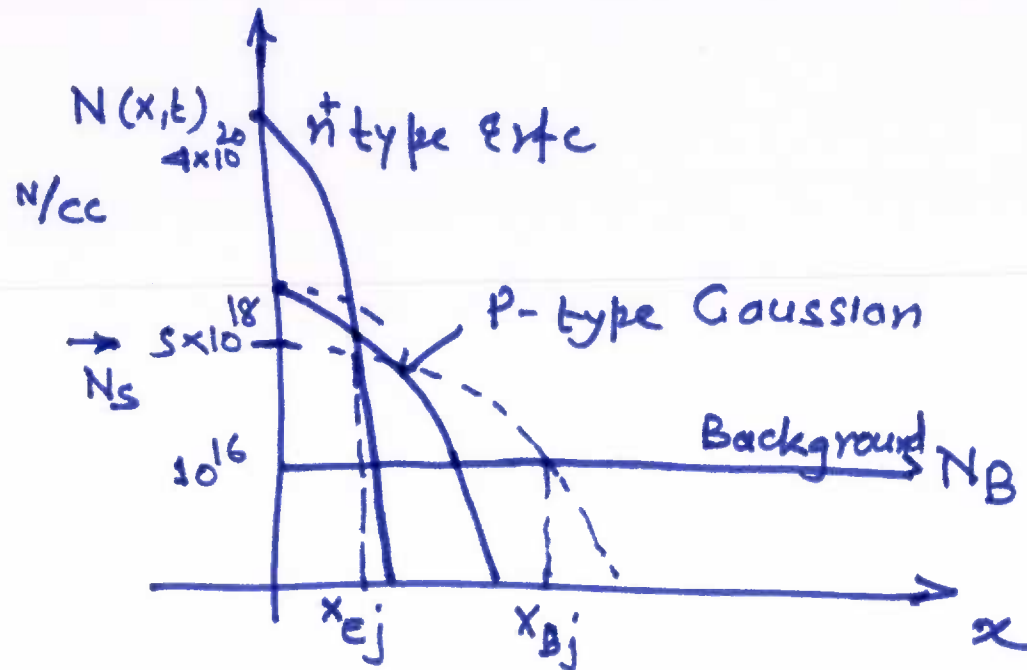
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Successive Diffusion



$$(Dt)_{eff} = \sum D_1 t_1 + D_2 t_2 + D_3 t_3 + \dots$$

① Substrate n-type = $10^{16}/cc$

② Base Diffusion

Predeposition $\rightarrow D_1 t_1$

Drive-In $\rightarrow D_2 t_2$

③ Emitter Diffusion

Only Predeposition $\rightarrow D_3 t_3$

$$\therefore \text{Base Diffusion } Dt_{eff} = D_1 t_1 + D_2 t_2 + D_3 t_3$$

Junction Formation in Two Step Diffusion

We have

$$N(x, t_1, t_2) = \frac{2N_{o1}}{\pi} \sqrt{\frac{D_1 t_1}{(Dt)_{eff}}} \exp\left(-\frac{x^2}{4(Dt)_{eff}}\right)$$

At $x=0$ $N(x, t_1, t_2 \dots) = N_s$ Surface Conc.

$$\text{or } N_s = \frac{2N_{o1}}{\pi} \sqrt{\frac{D_1 t_1}{(Dt)_{eff}}}$$

$$\text{or } N(x, t_1, \dots) = N_s \exp\left(-\frac{x^2}{4(Dt)_{eff}}\right)$$

At Junction of B-C (as shown) in Fig

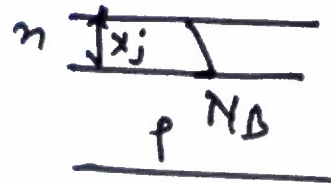
$$N(x_j, t_1, t_2) = N_B$$

$$\therefore N_B = N_s \exp\left(-\frac{x_j^2}{4(Dt)_{eff}}\right)$$



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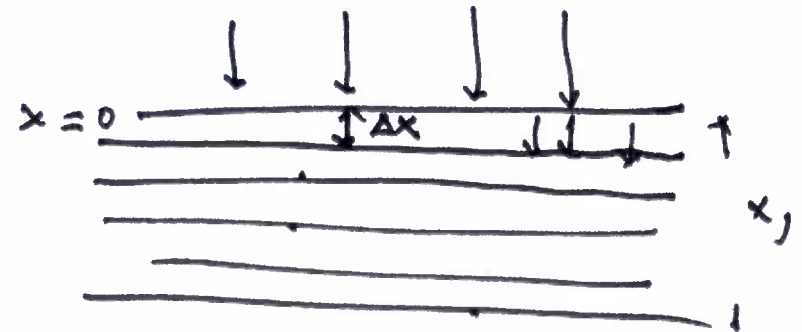
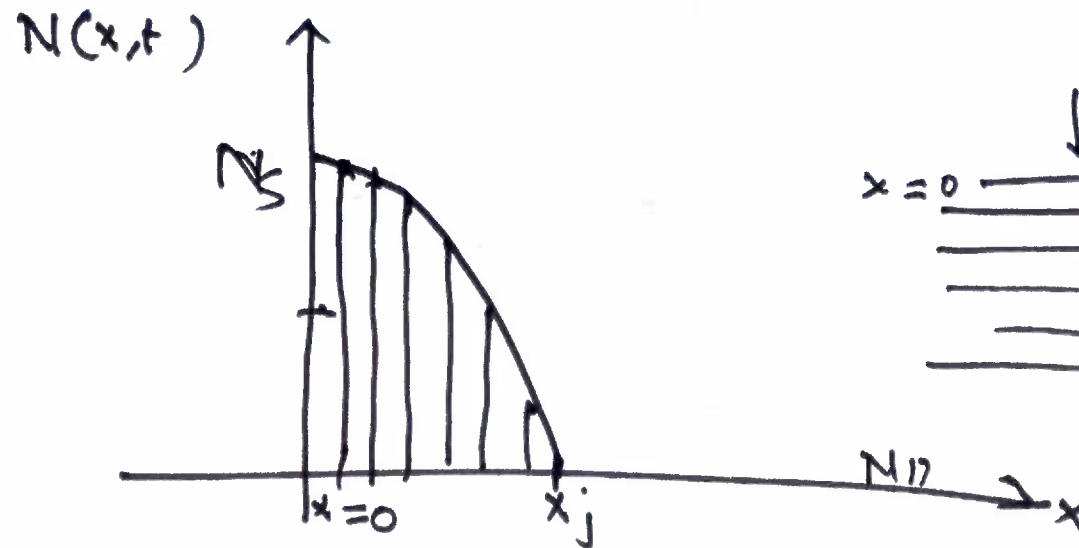
$$\text{or } x_j^2 = 4(Dt)_{\text{eff}} \ln \left(\frac{N_s}{N_b} \right)$$

$$\text{or } x_{jBC} = 2 \sqrt{(Dt)_{\text{eff}}} \sqrt{\ln \left(\frac{N_s}{N_b} \right)}$$



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Earlier we had defined Sheet Resistance R_s for uniformly doped Semiconductor bar with dimension $L \times w \times t$ as

$$R_s = \frac{\rho}{t} \quad \& \quad \rho = \frac{1}{q \mu_n n} \quad (\text{n-type})$$

$$\therefore R = R_s \left(\frac{L}{w} \right)$$

$$N(x, t) = n(x)$$

In a graded semiconductor with carrier conc profile of $n(x)$ or $p(x)$, we can get R_s knowing x_j .

$$R_s = \frac{1}{\int_0^{x_j} \sigma(x) dx}$$

$$\text{where } \sigma(x) = q \mu (N(x) - N_B)$$

$$R_s x_j = \frac{x_j}{\int_0^{x_j} q \mu [N(x) - N_B] dx}$$

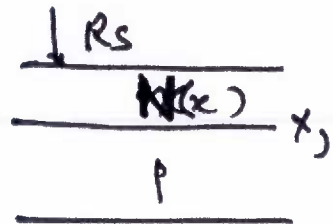


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$$\text{or } \frac{1}{R_s x_j} = q \mu (n/p) \frac{1}{x_j} \int_0^{x_j} [N(x) - N_B] dx$$

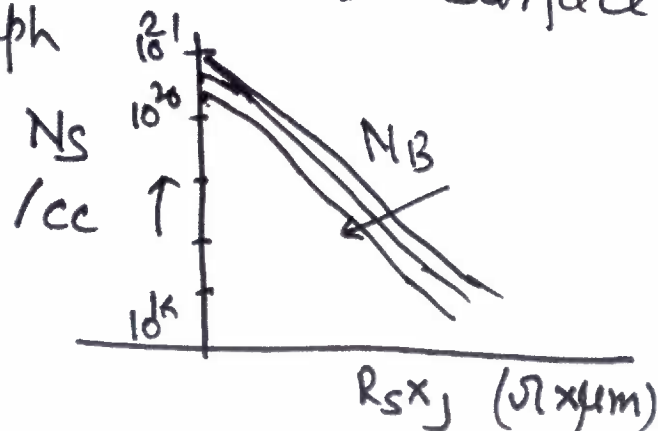
Hence we can plot $(R_s x_j)$ versus $N(x)$



We can monitor R_s at the surface and also can get value of x_j by Angle Lapping technique.

$$\text{Then } (R_s x_j)^{-1} = \frac{1}{x_j} q \mu [N(x=0) - N_B] dx$$

But $N(x)$ at $x=0 \rightarrow N_s$ Surface Conc. Hence we can have graph



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Example

For a case of Const. Source Diffusion

$$N(x) = N_s \operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right) \text{ where } N_s = N_0$$

While for a case of Limited Source Diffusion

$$N(x) = 2N_0 \sqrt{\frac{D_1 t_1}{D_2 t_2}} \exp\left(-\frac{x^2}{4D_2 t_2}\right)$$

$$\text{Where } N_s = 2N_0 \sqrt{\frac{D_1 t_1}{D_2 t_2}}$$

Hence by knowing R_s and x_j at the surface after Diffusion, we can get N_s for given substrate conc.



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Measurement of R_s

Four Point Probe

$$\rho_s = \frac{V}{I} \times 2\pi s \text{ (ohms-cm)}$$

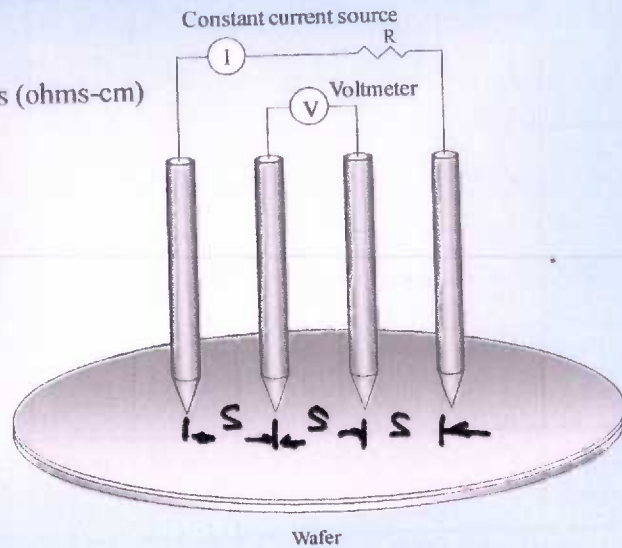


Figure 7.3

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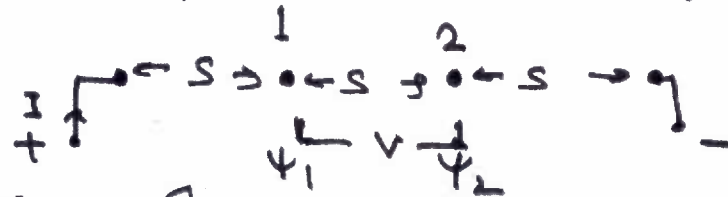
Analysis:

We take a case of 3 Points separated by distance r_1 & r_2



$$\therefore \psi_p = \frac{IR_s}{2\pi} \ln \frac{r_2}{r_1} + A$$

In Four Point ~~Probe~~ probe case, Probes are separated by distance s .



$$\therefore \psi_1 = \frac{IR_s}{2\pi} \ln \frac{s}{2s} + A$$

$$\psi_2 = \frac{IR_s}{2\pi} \ln \frac{2s}{s} + A$$

$$\therefore V = \psi_1 - \psi_2 = \frac{IR_s}{\pi} \ln 2$$

$$\therefore R_s = \frac{\pi}{\ln 2} \cdot \frac{V}{I}$$



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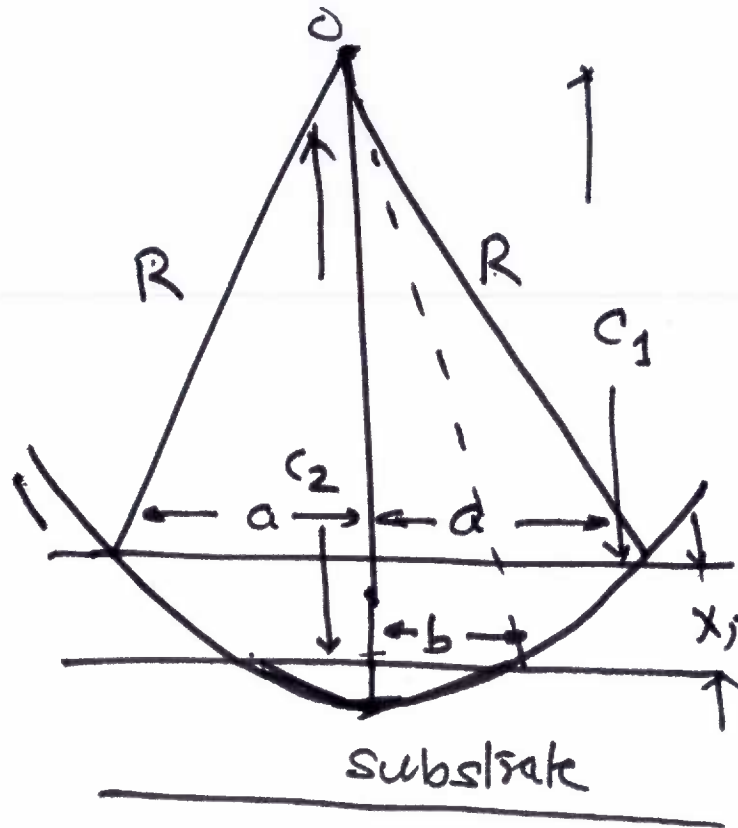
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Junction Depth Evaluation



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$$R^2 = a^2 + c_1^2$$

$$\text{or } c_1 = \sqrt{R^2 - a^2}$$

$$\text{Further } R^2 = b^2 + c_2^2$$

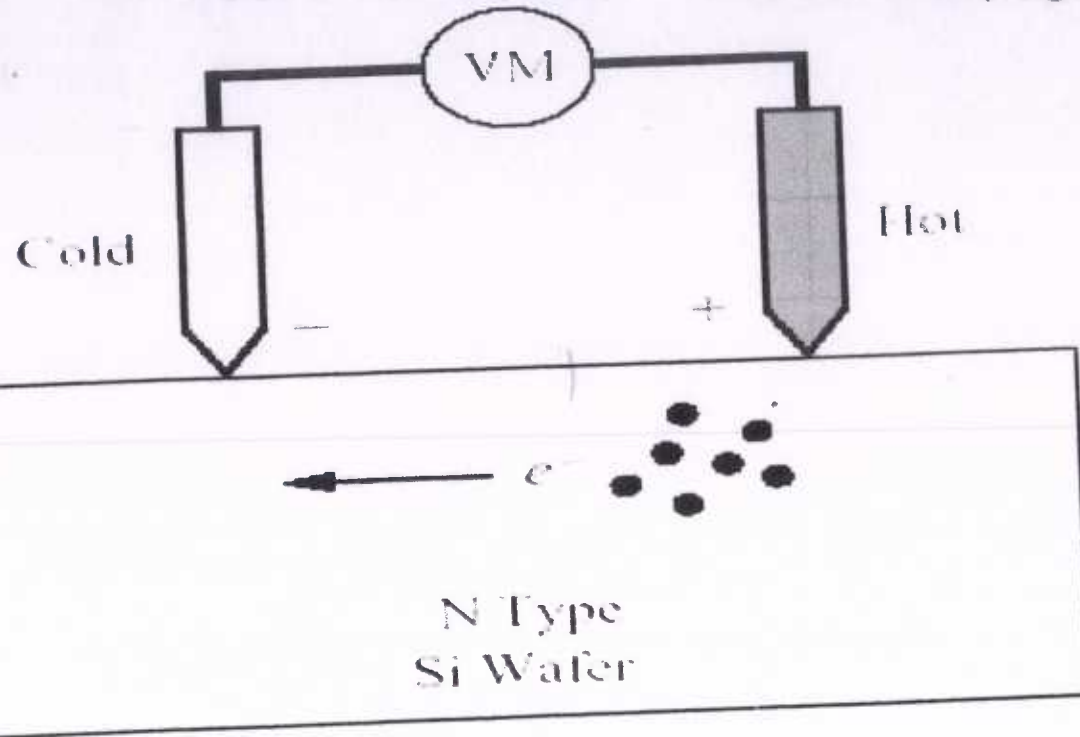
$$\text{or } c_2 = \sqrt{R^2 - b^2}$$

$$\therefore c_2 - c_1 = x_j = \sqrt{R^2 - b^2} - \sqrt{R^2 - a^2}$$

If we choose screw such that
 $R \gg a$ & $R \gg b$

$$x_j = c_2 - c_1 = R \left[\left(1 - \frac{b^2}{2R^2} \right) - \left(1 - \frac{a^2}{2R^2} \right) \right] = \frac{a^2 - b^2}{2R}$$

HOT PROBE MEASUREMENT



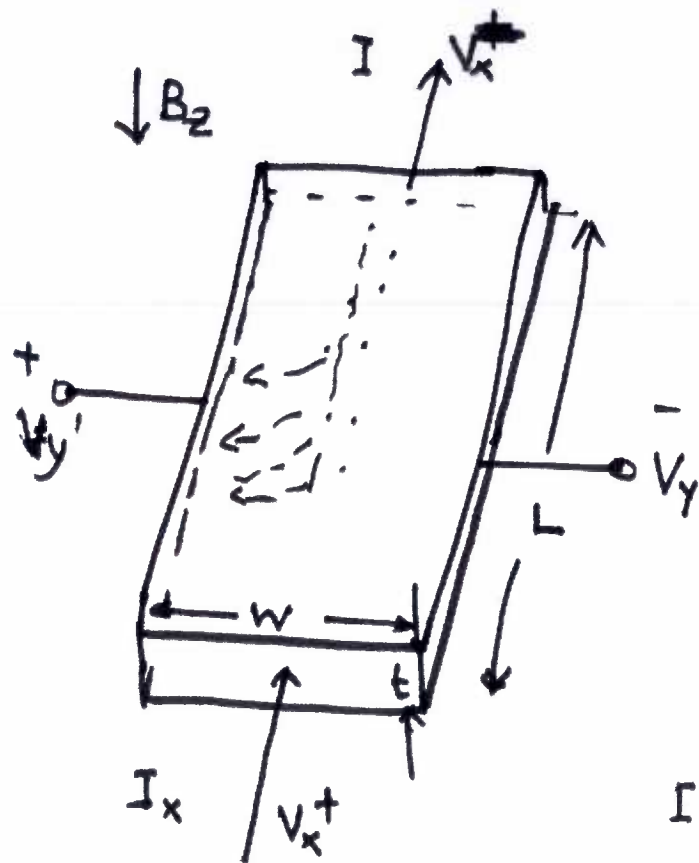
One can find type (n or p) of wafer using Hot Probe Technique.



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Hall Effect Measurement



We have

$$\frac{\rho L}{A} = R$$

With No Magnetic Field

$$R = \frac{V_x}{I_x}$$

$$\Delta \text{ Area} = w \cdot t$$

$$\therefore \rho = \frac{w \cdot t}{L} \frac{V_x}{I_x}$$

If Semiconductor is n -type, then

$$\rho = \frac{1}{q \mu_n n_0 + q \mu_p \frac{n_i^2}{n_0}} \approx \frac{1}{q \mu_n n_0}$$



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We now apply Magnetic Field B_z along z-direction.

The current direction shown (I_x), means electrons move from V_x^- to V_x^+ .

They are accelerated due to electric Field $E_x (= \frac{V_x}{L})$ and experience a force $q \frac{V_x}{L}$

Due to Orthogonal Magnetic Field B_z , electrons experience Lorentz Force $= q v_y B_z$ (v_y is velocity along y direction)

In equilibrium

$$q v_y B_z + q E_x = 0$$

$$\text{or } E_x = - v_y B_z = + B_z \frac{I_x}{q W \cdot t n_0}$$

$I = q n v$
 $n = \text{electron Conc}$

$$\text{Hall Voltage } V_y = - \int_0^w E_x dy$$

$$= B_z \frac{I_x}{q t n}$$

We define Hall Coeff $R_H = \frac{t V_y}{B_z I_x}$

$$\pm q R_H = \frac{1}{n} \quad \therefore n, p = \pm \frac{1}{q R_H}$$

We know $\rho_n = \frac{1}{q \mu_n n_0} = \sigma^{-1}$

$$\therefore \mu_n = \frac{R_H}{\rho} = |R_H| \cdot \sigma_n$$

$$\mu_p = |R_H| \sigma_p$$



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$R_H = +$ for Holes
 $R_H = -$ for Electrons

Example for Two Step Diffusion:

1. P type doping into n-type Silicon

2 $N_d = 4 \times 10^{16} / \text{cc}$

3 Predeposition: $T_1 = 950^\circ\text{C}$ for time $t_1 = 10 \text{ min}$

From Diffusion Const. - Temp. Graph: $D_1(950^\circ) \approx 10^{-14} \text{ cm}^2/\text{sec}$

$$1000/1223^\circ\text{K} = 0.8176$$

$$t_1 = 600 \text{ sec.}$$

$$\therefore D_1 t_1 = 6 \times 10^{-12} \text{ cm}^2$$

$$\text{or } \sqrt{D_1 t_1} = 2.45 \times 10^{-6} \text{ cm.}$$

a Drive-In after Predeposition in O_2 ambient (+ N_2)

for 60 minutes at $T_2 = 1100^\circ\text{C} = 1373^\circ\text{K}$

Hence $t_2 = 3600 \text{ sec}$ and $D(T_2) = D_2 = 4 \times 10^{-13} \text{ cm}^2/\text{sec}$

$$\therefore D_2 t_2 = 1.44 \times 10^{-9} \text{ cm}^2, \sqrt{D_2 t_2} = 3.79 \times 10^{-5} \text{ cm}$$



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5. During Predeposition, Temperature was 950°C .

The active Solid Solubility $N_0 = 3 \times 10^{20}/\text{cc}$
for ~~Phosphorous~~. Boron



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6. Evaluate R_s value.

7. From $R_s x_j - N_{\text{surface}}$ graph for $N_B = 4 \times 10^{16}/\text{cc}$,
and for Gaussian Profile, we can get $R_s x_j$ if
 $N_{\text{surf.}}$ is known. If we also know x_j , then we
know R_s .

8. For 2-step Diffusion

$$\begin{aligned} N(x, t_1, t_2) &= \frac{2N_0}{\pi} \sqrt{\frac{D_1 t_1}{D_2 t_2}} \exp\left[-\frac{x^2}{4D_2 t_2}\right] \\ &= N_{\text{surf}} \exp\left[-x^2/4D_2 t_2\right] \end{aligned}$$

$$\therefore N_{\text{surf}} = \frac{2N_{01}}{\pi} \sqrt{\frac{D_1 t_1}{D_2 t_2}}$$

$$= \frac{2 \times 3 \times 10^{20}}{3.14} \sqrt{\frac{6 \times 10^{-12}}{1.44 \times 10^{-9}}}$$

$$= \frac{6 \times 10^{20}}{3.14} \cdot \frac{2.45 \times 10^{-6}}{3.79 \times 10^{-5}} = 1.24 \times 10^{19} / \text{cc}$$

9. Further $X_j = 2 \sqrt{D_2 t_2} \left[\ln \frac{N_{\text{surf}}}{N_0} \right]^{1/2}$

$$= 2 \times 3.79 \times 10^{-5} \left[\ln \frac{1.24 \times 10^{19}}{4 \times 10^{16}} \right]$$

$$\approx 2 \times 3.79 \times 10^{-5} \times 5.74$$

$$= 4.35 \mu\text{m}.$$



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10. From $R_S X_j$ vs. N_{surf} graph, we obtain

$$R_S X_j = 200 \text{ } \Omega \text{cm} \quad \text{for } N_B = 4 \times 10^{16} / \text{cc} \text{ \& } N_{surf} = 1.24 \times 10^{19} / \text{cc}$$

$$\therefore R_S = \frac{200}{4.35} \text{ ohms}/\square$$

$$R_S \approx 46 \text{ ohms}/\square$$



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