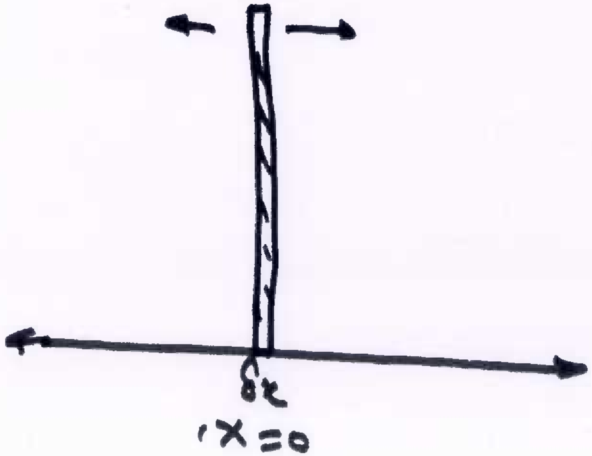
IInd case :- Limited Source Diffusion

In this case fixed amount of Impurities reside at the Surface and then Temperature-Time cycle starts.

From Diffusion Equation we have:

$$\frac{d^2 N(x,s)}{dx^2} - \frac{s\, N(x,s)}{D} = \frac{N(x, t \leq 0)}{D} \quad \left(= \frac{Q}{sD} \right)$$



Let us have a Sheet charge with DOSE Q (no/area) as fixed quality and is like a δ function at x=0

For this case Initial Cond^n is

$$N(x, t \leq 0^-) = Q \cdot \delta$$

We have solution of Diffusion Equation as for earlier constant source case as

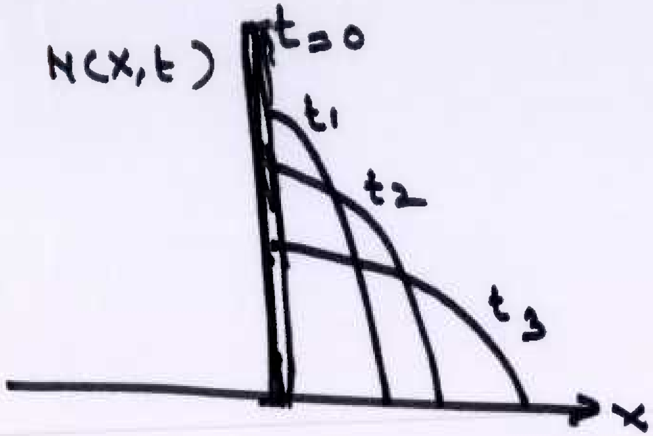
$$N(x,s) = B(s) \exp\left\{-\left(\frac{s}{D}\right)^{1/2} x\right\}$$

$$\mathcal{L}\left[N(x, t \leq 0^-)\right] = \frac{Q}{s}$$

$$\Rightarrow \therefore N(x,s) = \frac{Q}{\sqrt{s \cdot D}} \exp\left[-\left(\frac{s}{D}\right)^{1/2} x\right]$$

$$N(x,t) = \frac{Q}{\sqrt{\pi D t}} \exp\left(-\frac{x^2}{4Dt}\right)$$

This is essentially a Gaussian Profile



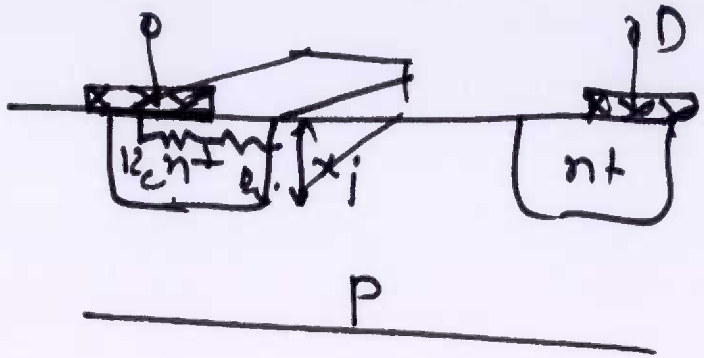
$t_3 > t_2 > t_1$

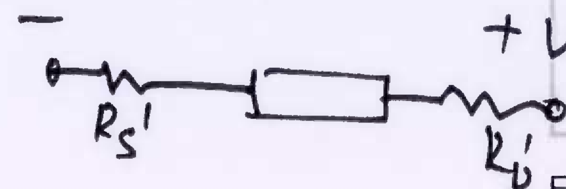
Some interesting points to note :

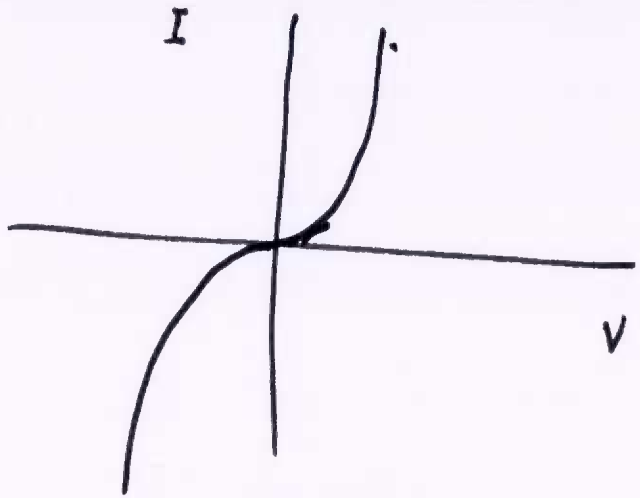
1. Since Impurity Dose was provided only once, it remains Constant

2. However subsequent time cycles (at 'A Temperature') spreads the Impurity Profile as

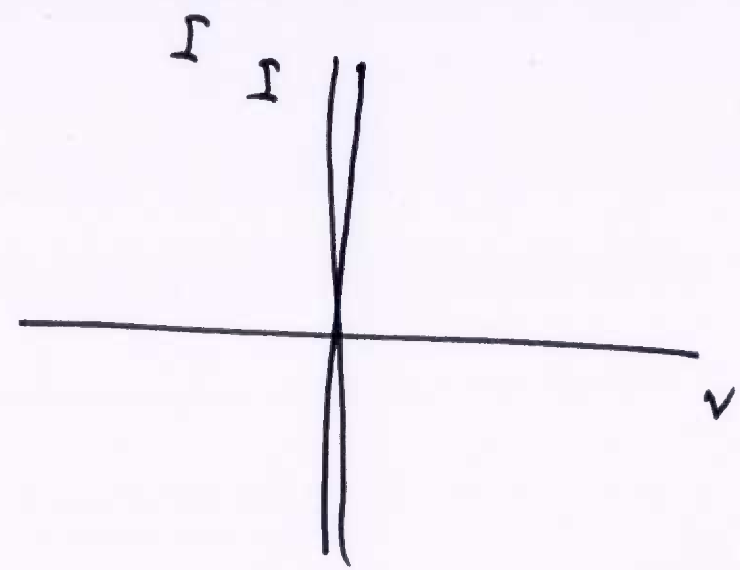
$$\int_0^{x'} N(x)\,dx = Q$$







Schottky
Diode



Since $N(x,t) = \frac{Q}{\sqrt{\pi D t}} \exp\left[-\left(\frac{x}{2\sqrt{Dt}}\right)^2\right]$

We conclude that $\sqrt{Dt}$ Function decides $N(x,t)$

Next we look erfc profile again.

Evaluation of Dose of Impurities in Constant Source Diffusion case

In this case $N(x,t) = N_0 \operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)$

We define $y = \frac{x}{2\sqrt{Dt}}$ $\therefore$ $\frac{dy}{dx} = \frac{1}{2\sqrt{Dt}}$

From Fick's First Law

$$j(x,t) = -D \frac{\partial N}{\partial x}$$

At the Surface ($x=0$) Flux density is

$$j(0,t) = -D \left.\frac{\partial N}{\partial x}\right|_{x=0}$$

From erfc profile we get

$$\left.\frac{dN}{dx}\right|_{x=0} = N_0 \frac{d}{dx}\left[\operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)\right]$$

$$\frac{d}{dx} = \frac{d}{dy} \cdot \frac{dy}{dx}$$

$$= N_0 \cdot \frac{d}{dy}\left[\operatorname{erfc}(y)\right] \cdot \frac{1}{2\sqrt{Dt}}$$

$$= \frac{N_0}{2\sqrt{Dt}} \frac{d}{dy}\left(1 - \operatorname{erf}(y)\right)$$

$$= -\frac{N_0}{2\sqrt{Dt}} \frac{d}{dy}\left[\operatorname{erf}(y)\right]$$

$$= -\frac{N_0}{2\sqrt{Dt}} \cdot \frac{2}{\sqrt{\pi}} \cdot \exp(-y^2)$$

or $J(0,t) = + \frac{N_0 D}{\sqrt{\pi D t}} \exp(-y^2) \Big|_{y=0}$

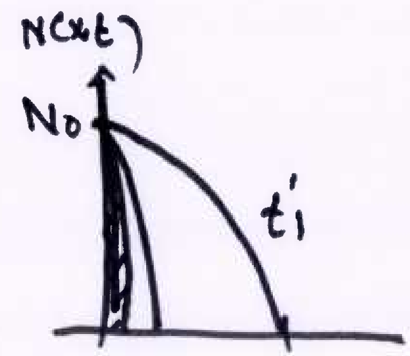
$= + N_0 \sqrt{\frac{D}{\pi t}}$

Now Dose $Q = \int_0^{t_1} [J(0,t)]\, dt$

where t_1 is the time for Diffusion at temperature T_1. At T_1, $D = D(T_1) = D_1$

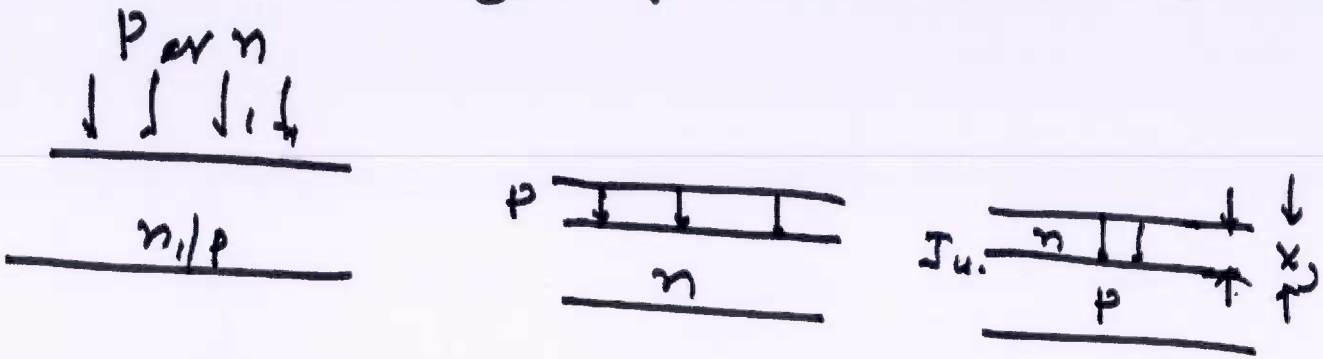
$\therefore \; Q_{T,t_1} = N_0 \sqrt{\frac{D_1}{\pi}} \left[\int_0^{t_1} t^{-1/2}\, dt \right]$

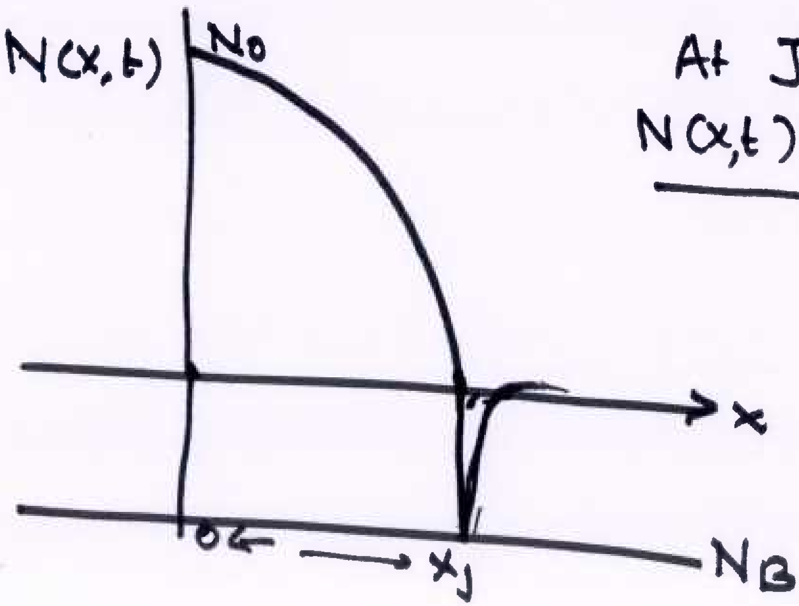
$= 2 N_0 \sqrt{\frac{D_1 t_1}{\pi}}$



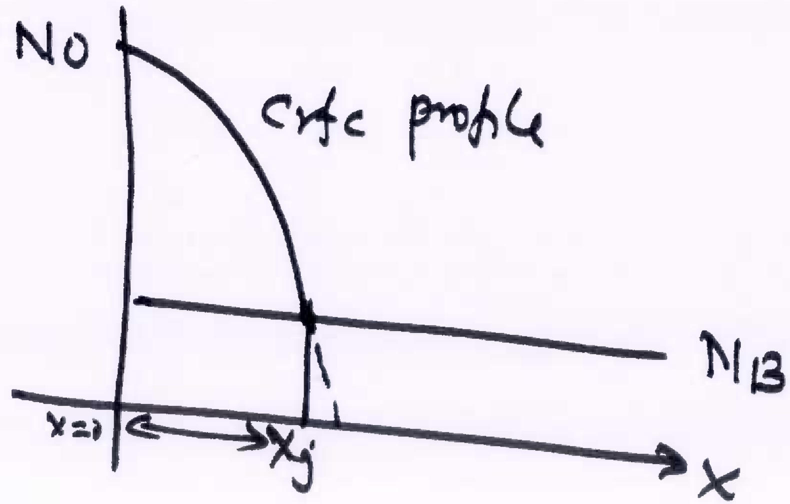
Junction Formation:

Starting substrate has base concentration as N_B.
It is one type of semiconductor (n or p).





At Junction
$N(x,t) - N_B = 0$



Let us take an exple of Diffusion of n-type of Impurities in p-type Substrate with conc. N_B. As n-type impurities enter p-wafer, electrons conc. increases near surface and reduces as Profile shows.

At $x = x_j$, n-type dopants = p-type doping (Compensation)

For Constant Source Diffusion

$$N(x,t) = N_0 \, \text{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)$$

If Diffusion is performed at Temperature T_1, then

$D(T_1) = D_1$ and $t = t_1$, time of diffusion.

$$\therefore Q(T_1, t_1) = 2N_0 \sqrt{\frac{D_1 t_1}{\pi}}$$

At Junction, net conc. is zero. Hence we say incoming Impurity conc is exactly same as N_B

$$\therefore \quad N(x_j, t_1) = N_B$$

$$\text{or} \quad N_0 \, \text{erfc}\left(\frac{x_j}{2\sqrt{D_1 t_1}}\right) = N_B$$

$$\text{or} \quad x_j = 2\sqrt{D_1 t_1} \, \text{erfc}^{-1}\left(\frac{N_B}{N_0}\right)$$

In specific case, $T_1 = 1000°C$ and Diffusion of Phosphorous takes place in p-doped wafer with conc $N_B = 5 \times 10^{15}/cc$ for total time of 60 minutes (3600 seconds)

At $T = T_1$:— $N_0 = 3 \times 10^{20}/cc$ for Phosphorous
& $D_1 = 3 \times 10^{-14}$ cm²/sec

$\therefore \sqrt{D_1 t_1} = \sqrt{3\times10^{-14}\times3600} = 1.04\times10^{-5}$ cm

Further $\text{erfc}^{-1}\left(\frac{N_B}{N_0}\right) = \text{erfc}^{-1}\left(\frac{5\times10^{15}}{3\times10^{20}}\right)$

$= \text{erfc}^{-1}(1.66\times10^{-5}) = x$

As $\text{erfc}(x) = 1 - \text{erf}(x)$.

~~Let us say erf = 1.66×10^{-5}~~

Let us define $\text{erfc}\, x = 1.66\times10^{-5}$

or $\text{erf}\, x = 1 - 1.6\times10^{-5}$

$= 0.999984$

$\therefore x = \text{erf}^{-1}(0.999984)$

$= 3.06$

$\therefore x_j = 2\sqrt{D_1 t_1}\cdot 3.06$

$x_j = 2\times1.04\times10^{-5}\times3.06$

or $x_j = 0.636\ \mu m$

In Practice, the Diffusion of Impurities is a Two-Step Process

(i) Predeposition (ii) Drive-In

(i) Predeposition :

Here we perform Constant Source diffusion at Temperature T_1 for time t_1

$$\therefore D_1 = D_1(T_1), \quad N_{o1} = N_o(T_1)$$

$$\therefore N(x, t_1) = N_{o1} \operatorname{erfc}\left(\frac{x}{2\sqrt{D_1 t_1}}\right)$$

And after time t_1, the dose of Impurities $Q(t_1)$ at Surface

$$= 2 N_{o1} \sqrt{\frac{D_1 t_1}{\pi}}$$

(ii) After Predeposition step, source of Impurities are stopped. Then the wafers are reintroduced in the furnace at Temperature T_2 for time t_2 this process is continued. This process is called Drive-In.

At T_2, $D_2(T_2) = D_2$, $\therefore Dt = D_2 t_2$

In Drive-In, no new impurities are introduced, but available ones from Predeposition are Re-distributed (Q dose). This is the case of Limited Source Diffusion giving Gaussian Profile:

$$N(x, t_1, t_2) = \frac{Q}{\sqrt{\pi D_2 t_2}} \exp\left(-\frac{x^2}{4 D_2 t_2}\right)$$

or $$N(x, t_1, t_2) = \frac{2 N_{01}}{\pi} \sqrt{\frac{D_1 t_1}{D_2 t_2}} \exp\left(-\frac{x^2}{4 D_2 t_2}\right)$$

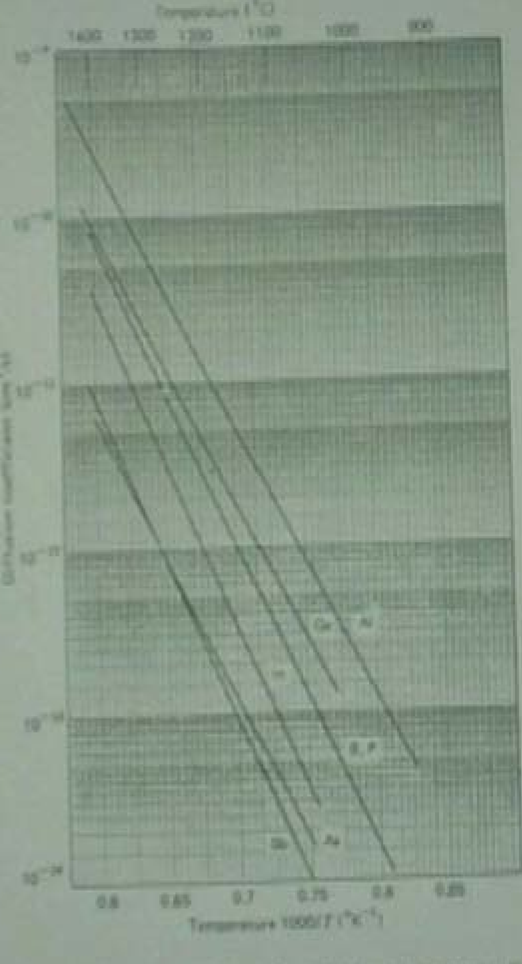


Fig. 4.7 Average diffusion coefficients for substitutional diffusers in silicon.

formed. This experimental technique essentially duplicates the fabrication
... Consequently, the results are reasonably accurate

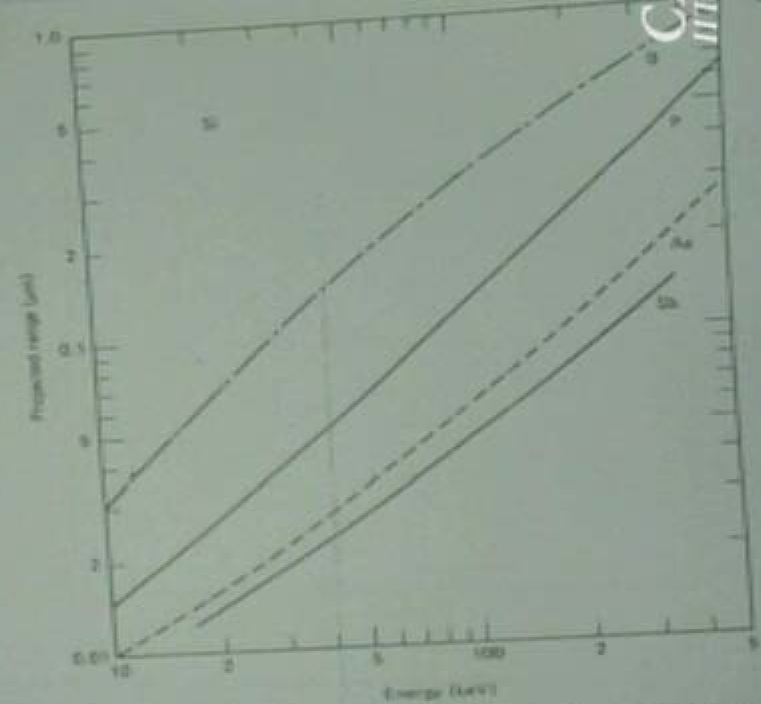
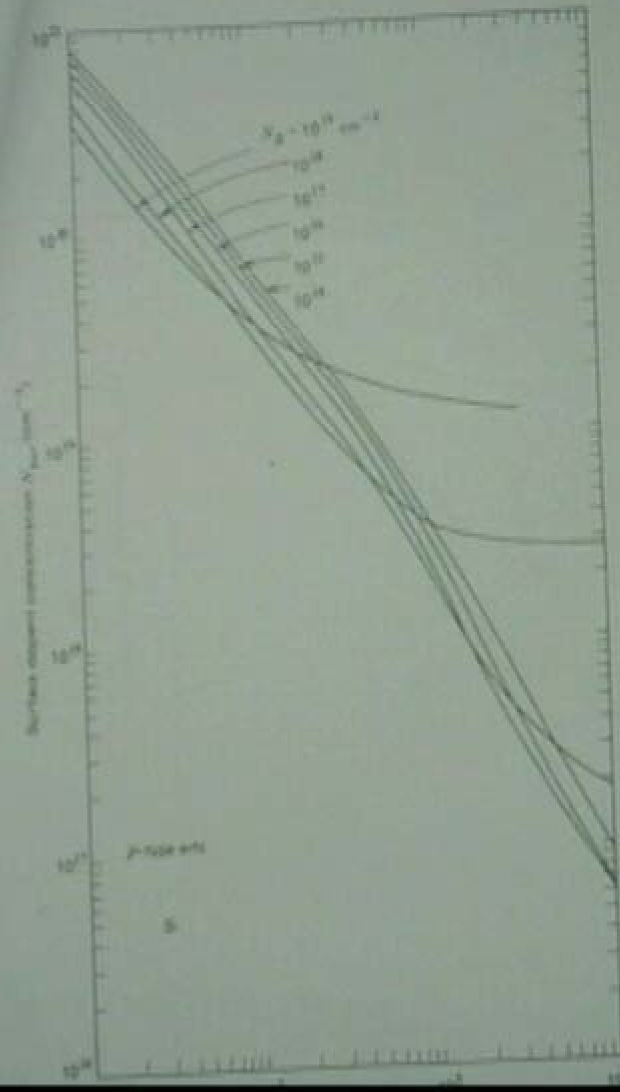


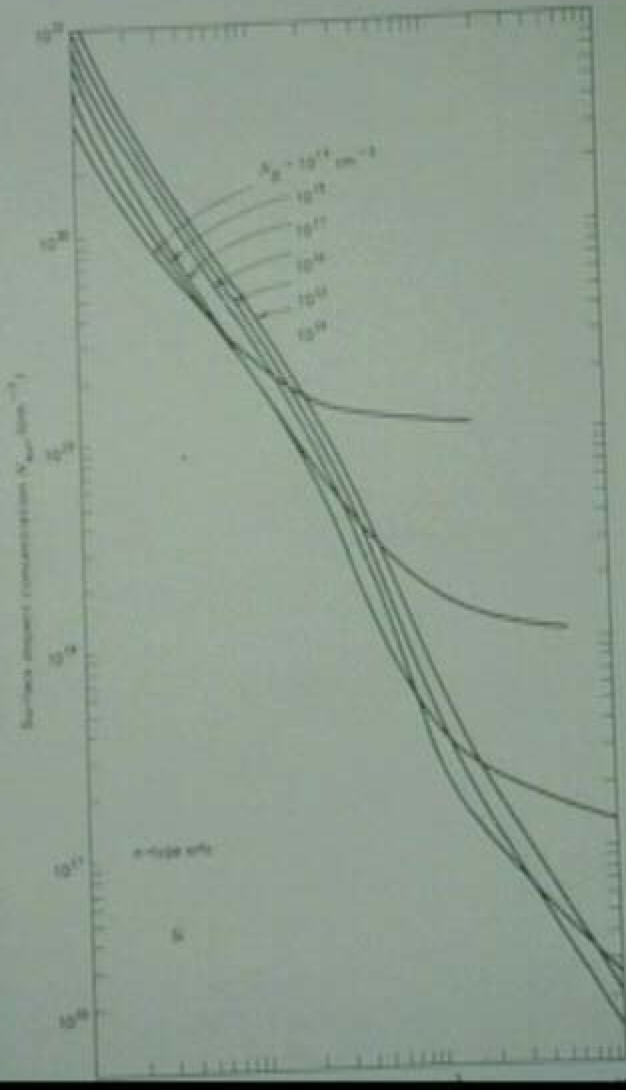
Fig. 6.12 Projected range of boron, phosphorus, arsenic, and antimony
Adapted from Gibbons, Johnson, and Mylroie [9].



ay

Slide

Q d



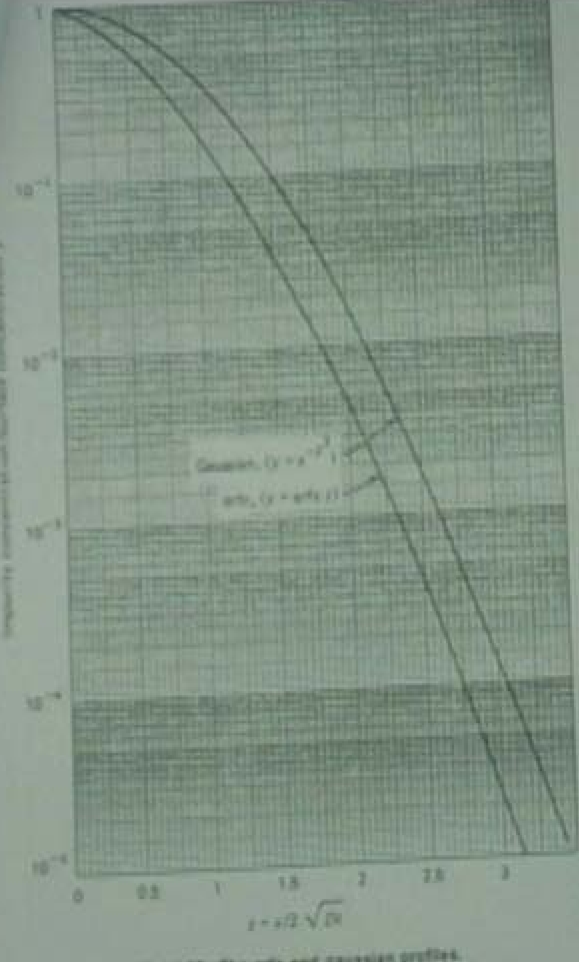
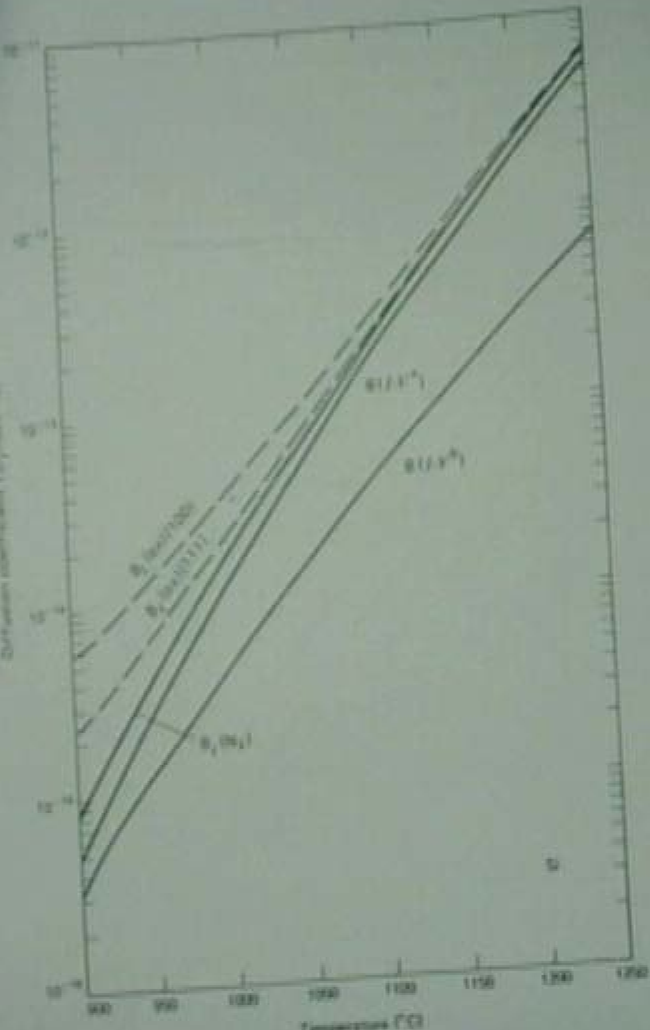


Fig. 4.17 The erfc and gaussian profiles.

4.5.1.2 Diffusion from a Limited Source

Here a finite quantity of the diffusing matter is first placed on the wafer and it is assumed that all of



(a)

Fig. 4.8 Diffusivities of boron in silicon. From R. Colclaser, *Microelectronics: Pro-*
... with permission of the publisher, John

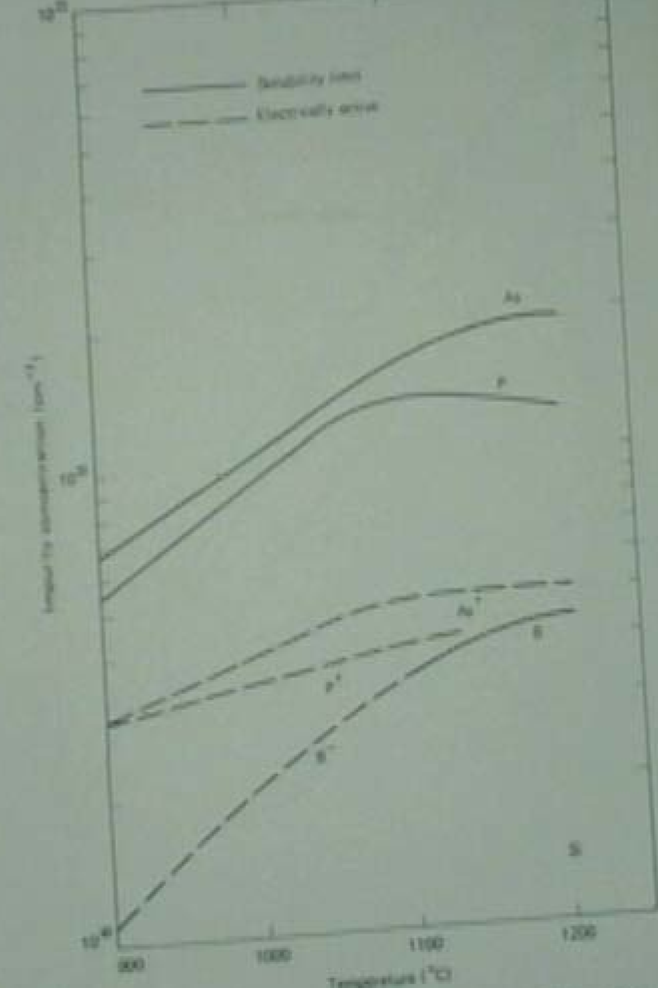


Fig. 4.11 Solubility limits for boron, arsenic, and phosphorus. From R. Colclaser, *Microelectronics: Processing and Device Design* [10], 1980. Reprinted with permission of the publisher, John Wiley & Sons.

Table A.1 Error Function erf z*

z	erf(z)	z	erf(z)	z	erf(z)	z	erf(z)
0.00	0.000 000	0.50	0.520 500	1.00	0.842 701	1.50	0.966 105
0.01	0.011 283	0.51	0.529 244	1.01	0.846 810	1.51	0.967 277
0.02	0.022 565	0.52	0.537 899	1.02	0.850 838	1.52	0.968 413
0.03	0.033 841	0.53	0.546 464	1.03	0.854 784	1.53	0.969 516
0.04	0.045 111	0.54	0.554 939	1.04	0.858 650	1.54	0.970 586
0.05	0.056 372	0.55	0.563 323	1.05	0.862 436	1.55	0.971 623
0.06	0.067 622	0.56	0.571 616	1.06	0.866 144	1.56	0.972 628
0.07	0.078 858	0.57	0.579 816	1.07	0.869 773	1.57	0.973 603
0.08	0.090 078	0.58	0.587 923	1.08	0.873 326	1.58	0.974 547
0.09	0.101 281	0.59	0.595 936	1.09	0.876 803	1.59	0.975 462
0.10	0.112 463	0.60	0.603 856	1.10	0.880 205	1.60	0.976 348
0.11	0.123 623	0.61	0.611 681	1.11	0.883 533	1.61	0.977 207
0.12	0.134 758	0.62	0.619 411	1.12	0.886 788	1.62	0.978 038
0.13	0.145 867	0.63	0.627 046	1.13	0.889 971	1.63	0.978 843
0.14	0.156 947	0.64	0.634 586	1.14	0.893 082	1.64	0.979 622
0.15	0.167 996	0.65	0.642 029	1.15	0.896 124	1.65	0.980 376
0.16	0.179 012	0.66	0.649 377	1.16	0.899 096	1.66	0.981 105
0.17	0.189 992	0.67	0.656 628	1.17	0.902 000	1.67	0.981 810
0.18	0.200 936	0.68	0.663 782	1.18	0.904 837	1.68	0.982 493
0.19	0.211 840	0.69	0.670 840	1.19	0.907 608	1.69	0.983 153
0.20	0.222 703	0.70	0.677 801	1.20	0.910 314	1.70	0.983 790
0.21	0.233 522	0.71	0.684 666	1.21	0.912 956	1.71	0.984 407
0.22	0.244 296	0.72	0.691 433	1.22	0.915 534	1.72	0.985 003
0.23	0.255 023	0.73	0.698 104	1.23	0.918 050	1.73	0.985 578
0.24	0.265 700	0.74	0.704 678	1.24	0.920 505	1.74	0.986 135
0.25	0.276 326	0.75	0.711 156	1.25	0.922 900	1.75	0.986 672
0.26	0.286 900	0.76	0.717 537	1.26	0.925 236	1.76	0.987 190
0.27	0.297 418	0.77	0.723 822	1.27	0.927 514	1.77	0.987 691
0.28	0.307 880	0.78	0.730 010	1.28	0.929 734	1.78	0.988 174
0.29	0.318 283	0.79	0.736 103	1.29	0.931 899	1.79	0.988 641
0.30	0.328 627	0.80	0.742 101	1.30	0.934 008	1.80	0.989 091
0.31	0.338 908	0.81	0.748 003	1.31	0.936 063	1.81	0.989 525
0.32	0.349 126	0.82	0.753 811	1.32	0.938 065	1.82	0.989 943
0.33	0.359 279	0.83	0.759 524	1.33	0.940 015	1.83	0.990 347
0.34	0.369 365	0.84	0.765 143	1.34	0.941 914	1.84	0.990 736
0.35	0.379 382	0.85	0.770 668	1.35	0.943 762	1.85	0.991 111
0.36	0.389 330	0.86	0.776 100	1.36	0.945 561	1.86	0.991 472
0.37	0.399 206	0.87	0.781 440	1.37	0.947 312	1.87	0.991 821
0.38	0.409 009	0.88	0.786 687	1.38	0.949 016	1.88	0.992 156
0.39	0.418 739	0.89	0.791 843	1.39	0.950 673	1.89	0.992 479
0.40	0.428 392	0.90	0.796 908	1.40	0.952 285	1.90	0.992 790
0.41	0.437 969	0.91	0.801 883	1.41	0.953 852	1.91	0.993 090
0.42	0.447 468	0.92	0.806 768	1.42	0.955 376	1.92	0.993 378
0.43	0.456 887	0.93	0.811 564	1.43	[illegible]	1.93	[illegible]