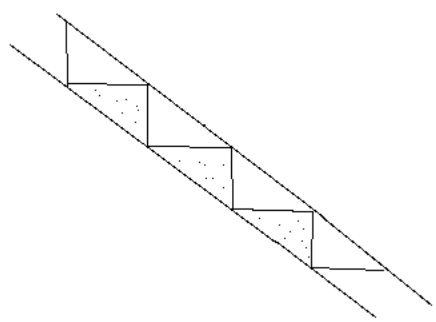
CSL 852, Computational Geometry: Practice Problems

Lower bounds on algebraic tree model

1. Prove an $\Omega(n \log h)$ lower bound for the 2D maxima problem where n and h are the input and output sizes respectively.
   Hint: Use a construction similar to the convex hull lower bound.

   Solution: In the construction below, there are total of n input points and exactly h points on the above line. Now look at the following relaxed decision version of the 2D Maxima problem :
   Are there exactly h maximal points out of the n given points ?
   For this, the remaining $n - h$ points must lie inside the wedges created by the lower line and the h points. Now since a point cannot pass from one wedge to another without going out of the wedges or the lower line, thus in the n-dimensional solution space, every different configuration of these $n - h$ points in h wedges is separated by a "No" instance i.e. a configuration of points returning answer "No" to the decision problem posted above. So the number of leaf nodes in the decision tree are atleast h^{n-h} and hence the time taken is $O(n \log h)$.
   Since this version of problem can be easily reduced to the normal 2D Maxima problem, thus the above lower bound applies to 2D maxima.

2. Prove an $\Omega(n \log h)$ bound for any convex hull algorithm in the algebraic decision tree model using the idea of reducing a more restricted class of instances (discussed in class).
   Hint: Use an n-gon that contains a circumscribed and inscribed disc and place the points in some selected regions.

3. Given two sets A and B such that $|A|+|B| = n$, prove an $\Omega(n \log n)$ bound to determine if $A \cap B = \Phi$.
   Hint: Consider an alternating sequence of points and argue that each order type corresponds to a component in the solution space.