CSL 852, Computational Geometry: Practice Problems

ε-nets, VC dimension and applications

1. An ε-sample of a range space $S = (X, \mathcal{R})$ is a subset $C \subset X$ such that for any range $r \in \mathcal{R}$, the following property is satisfied

$$|\frac{|X \cap r|}{|X|} - \frac{|C \cap r|}{|C|}| \leq \varepsilon$$

   Let us two color the elements X using $\{+1, -1\}$ and let χ denote coloring function. The *discrepancy* $disc(r)$ of a range $r \in \mathcal{R}$ is defined as $|\sum_{p \in X \cap r} \chi(p)|$, i,e, the imbalance of the coloring. The *discrepancy* of S is given by

$$\min_{\chi : X \to \{+1,-1\}} \max_{r \in \mathcal{R}} disc(r)$$

   which minimises the discrepancy over all possible colorings. Note that the discrepancy of a range space may be very expensive to compute.

   Show that if discrepancy for a set X of n elements can be bounded by $\tau(n)$, then there exists a $\tau(n)/n$-sample. You may assume that the range space has bounded VC dimension.