Advanced Topics in Optimization

Evolutionary Algorithms for Optimization and Search

D Nagesh Kumar, IISc

Optimization Methods: M8L5
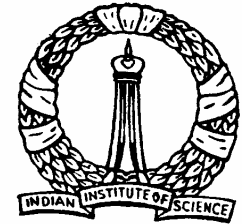
Introduction

➢ Real world optimization problems mostly involve complexities like discrete-continuous or mixed variables, multiple conflicting objectives, non-linearity, discontinuity and non-convex region

➢ Global optimum cannot be found in a reasonable time due to the large search space

➢ For such problems, existing linear or nonlinear methods may not be efficient

➢ Various stochastic search methods like simulated annealing, evolutionary algorithms (EA), hill climbing can be used in such situations
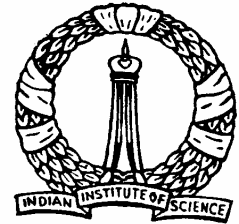
Introduction

➢ Among these techniques, the special advantage of EA's are

  ➢ Can be applied to any combination of complexities (multi-objective, non-linearity etc) and also

  ➢ Can be combined with any existing local search or other methods

➢ Various techniques which make use of EA approach

  ➢ Genetic Algorithms (GA), Evolutionary Programming, Evolution Strategy, Learning Classifier System etc.

➢ EA techniques operate mainly on a population search basis.

Objectives

➢ To discuss the basic concept of Evolutionary Algorithms (EA)

➢ To explain the basic steps for a simple EA like

    ➢ Parent Selection

    ➢ Cross over

    ➢ Mutation

➢ To discuss the advantages and disadvantages of EA

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Basic Concept of EA

- EAs start from a *population* of possible solutions (*called individuals*) and move towards the optimal one by applying the principle of Darwinian evolution theory i.e., *survival of the fittest*

- Objects forming possible solution sets to the original problem is called *phenotype*

- The encoding (representation) or the individuals in the EA is called *genotype*

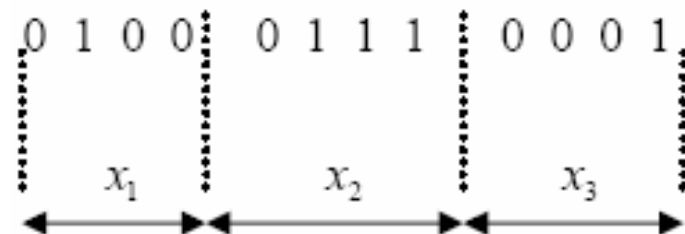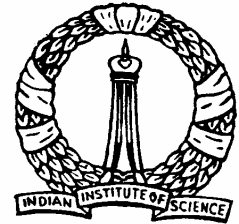- The mapping of phenotype to genotype differs in each EA technique

Basic Concept of EA …contd.

- In GA, variables are represented as strings of numbers (normally binary)

- Let the number of design variables be n

- Let each design variable is given by a string of length 'l

- Then the design vector will have a total string length 'nl'

- For example, if the string length be 4 for each design variable and there are 3 design variables,

$$x_1 = 4, x_2 = 7 \ and \ x_3 = 1$$

- Then the *chromosome* length is 12 as shown

$$\underbrace{0\ 1\ 0\ 0}_{x_1}\ \vdots\ \underbrace{0\ 1\ 1\ 1}_{x_2}\ \vdots\ \underbrace{0\ 0\ 0\ 1}_{x_3}$$
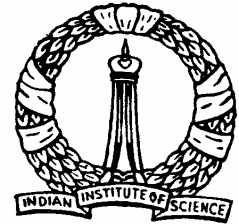
Basic Concept of EA …contd.

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$$0\ 1\ 0\ 0 \mid 0\ 1\ 1\ 1 \mid 0\ 0\ 0\ 1$$

$x_1 \qquad x_2 \qquad x_3$
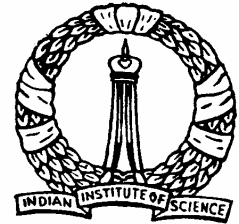
Basic Concept of EA …contd.

- An individual consists of a genotype and a fitness function

- *Fitness* Function:

    - Represents the quality of the solution

    - Forms the basis for selecting the individuals and thereby facilitates improvements

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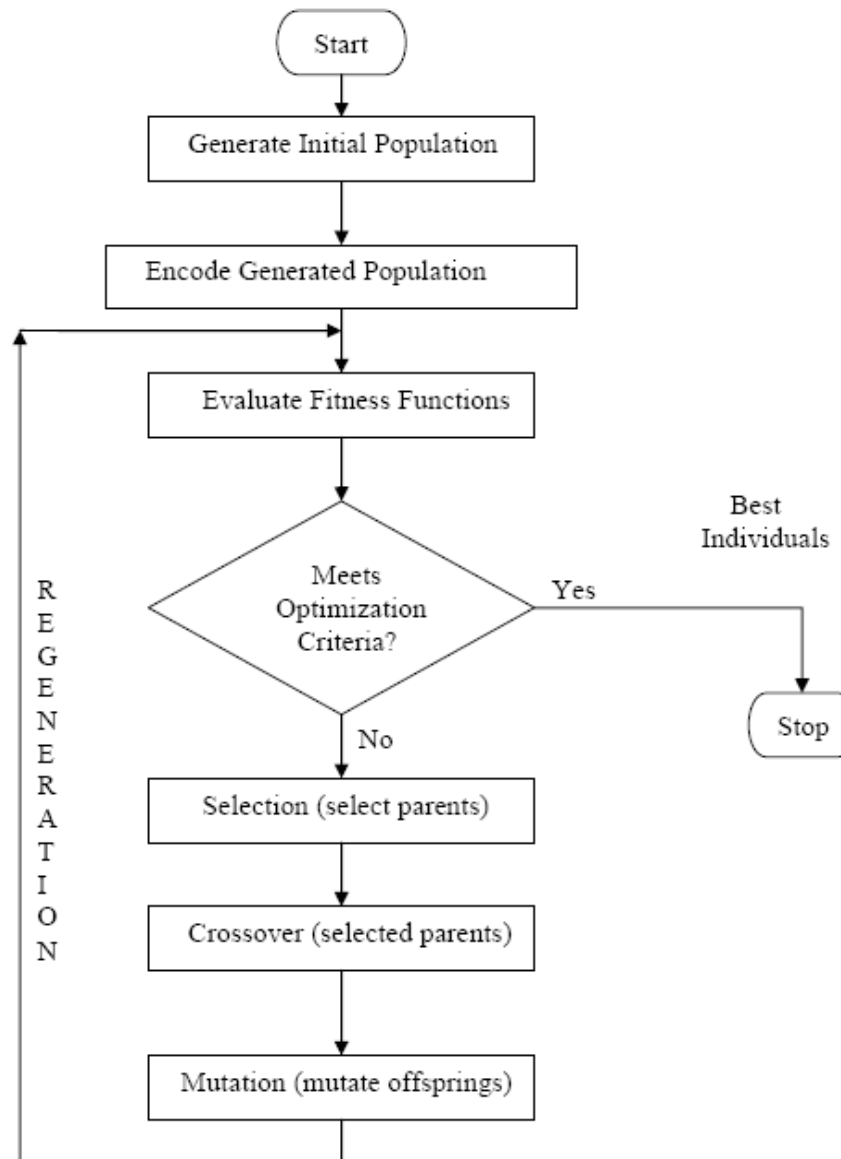# Basic Concept of EA …contd.

Pseudo code for a simple EA

$i = 0$

Initialize population P_0

Evaluate initial population

while (! termination condition)

{

    $i = i+1$

    Perform competitive selection

    Create population P_i from P_{i-1} by recombination and mutation

    Evaluate population P_i

}

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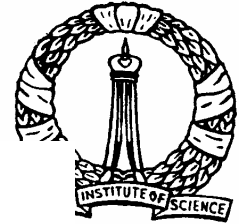
Basic Concept of EA …contd.

Flowchart indicating the steps of a simple genetic algorithm

Start

Generate Initial Population

Encode Generated Population

Evaluate Fitness Functions

Meets Optimization Criteria?

Yes → Best Individuals → Stop

No

Selection (select parents)

Crossover (selected parents)

Mutation (mutate offsprings)

REGENERATION

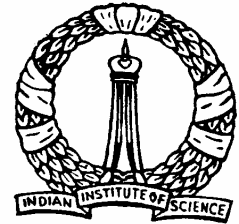D Nagesh Kumar, IISc   Optimization Methods: M8L5

Basic Concept of EA …contd.

- Initial population is randomly generated

- Individuals with better fitness functions from generation 'i' are taken to generate individuals of '$i+1$'th generation

- New population (*offspring*) is created by applying *recombination* and *mutation* to the selected individuals (*parents*)

- Finally, the new population is evaluated and the process is repeated

- The termination condition may be a desired fitness function, maximum number of generations etc

Parent Selection

+ Individuals are distinguished based on their fitness function value

+ According to Darwin's evolution theory the best ones should survive and create new offspring for the next generation

+ Different methods are available to select the best chromosomes

    + Roulette wheel selection, Rank selection, Boltzman selection, Tournament selection, Steady state selection

Parent Selection: Roulette wheel selection

- Each individual is selected with a probability proportional to its fitness value

- In other words, an individual is selected depending on the percentage contribution to the total population fitness

- Thus, weak solutions are eliminated and strong solutions survive to form the next generation

Parent Selection: Roulette wheel selection …contd.

Consider a population containing four strings shown

Candidate	Fitness value	Percentage of total fitness
1011 0110 1101 1001	109	28.09
0101 0011 1110 1101	76	19.59
0001 0001 1111 1011	50	12.89
1011 1111 1011 1100	153	39.43
Total	388	100

- Each string is formed by concatenating four substrings representing variables a, b, c and d. Length of each string is taken as four bits
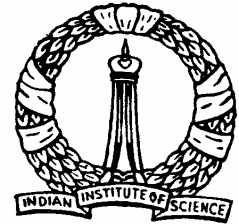
Parent Selection: Roulette wheel selection …contd.

- First column represents the possible solution in binary form

- Second column gives the fitness values of the decoded strings

- Third column gives the percentage contribution of each string to the total fitness of the population

- Thus, the probability of selection of candidate 1, as a parent of the next generation is 28.09%

- Probabilities of other candidates 2, 3, 4 are 19.59, 12.89 and 39.43 respectively

Parent Selection: Roulette wheel selection …contd.

- These probabilities are represented on a pie chart

- Then four numbers are randomly generated between 1 and 100

- The likeliness of these numbers falling in the region of candidate 2 might be once, whereas for candidate 4 it might be twice and candidate 1 more than once and for candidate 3 it may not fall at all

- Thus, the strings are chosen to form the parents of the next generation

- The main disadvantage of this method is when the fitnesses differ very much

- For example, if the best chromosome fitness is 90% of the entire roulette wheel then the other chromosomes will have very few chances to be selected

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Parent Selection: Rank Selection

- Population is ranked first

- Every chromosome will be allotted with one fitness corresponding to this ranking

- The worst will have fitness 1, second worst 2 etc. and the best will have fitness N (number of chromosomes in population)

- By doing this, all the chromosomes have a chance to be selected

- But this method can lead to slower convergence, because the best chromosomes may not differ much from the others

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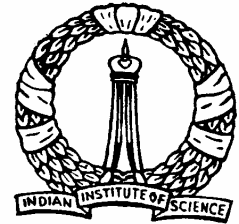
Parent Selection

- Through selection new individuals cannot get introduced into the population

- Selection cannot find new points in the search space

- New individuals are generated by genetically-inspired operators known are *crossover* and *mutation*.
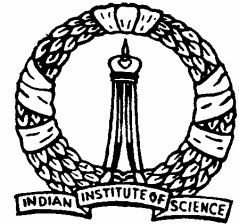
Crossover

- Crossover can be of either one-point or two-point scheme
- *One point crossover:* Selected pair of strings is cut at some random position and their segments are swapped to form new pair of strings
- Consider two 8-bit strings given by '10011101' and '10101011'
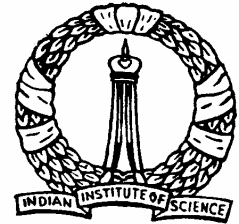- Choose a random crossover point after 3 bits from left

$$100 \mid 11101$$

$$101 \mid 01011$$

- Segments are swapped and the resulting strings are

$$10001011$$

$$10111101$$

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Crossover ...contd.

- *Two point crossover:* There will be two break points in the strings that are randomly chosen

- At the break-point the segments of the two strings are swapped so that new set of strings are formed
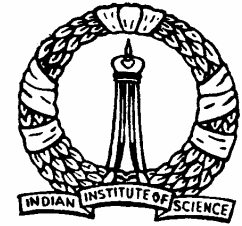
- If two crossover points are selected as

$$100 \mid 11 \mid 101$$

$$101 \mid 01 \mid 011$$

- After swapping both the extreme segments, the resulting strings formed are

$$10001101$$

$$10111011$$

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Mutation

- Mutation is applied to each child individually after crossover

- Randomly alters each gene with a small probability (generally not greater than 0.01)

- Injects a new genetic character into the chromosome by changing at random a bit in a string depending on the probability of mutation

- Example: *10111__0__11 is mutated as*

    10111__1__11

    Sixth bit '0' is changed to '1'

- In mutation process, bits are changed from *'1' to '0'* or *'0' to '1'* at the randomly chosen position of randomly selected strings

Advantages and Disadvantages of EA:

Advantages

➤ EA can be efficiently used for highly complex problems with multi-objectivity, non-linearity etc

➤ Provides not only a single best solution, but the 2nd best, 3rd best and so on as required.

➤ Gives quick approximate solutions

➤ Can incorporate with other local search algorithms

Advantages and Disadvantages of EA:

Disadvantages

➢ Optimal solution cannot be ensured on using EA methods

➢ Convergence of EA techniques are problem oriented

➢ Sensitivity analysis should be carried out to find out the range in which the model is efficient

➢ Implementation requires good programming skill

Thank You

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